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COSMIC RAY COLLISIONS IN SPACE

PART II — HIGH ENERGY GAMMA RAYS FROM COSMIC RAY COLLISIONS IN SPACE

by M. Lieber, S. N. Milford, and M. S. Spergel

Prepared under Contract No. NASw-699 *by*
GRUMMAN AIRCRAFT ENGINEERING CORPORATION
Bethpage, N. Y.

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JANUARY 1965

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FOREWORD

This document comprises Part II of the final report on Contract No. NASw-699, Cosmic Ray Collisions in Space. The complete report describes in detail the research carried out on this contract by the Geo-Astrophysics Section of the Research Department of Grumman Aircraft Engineering Corporation between July 3, 1963 and November 3, 1964. This work was performed under the technical cognizance of Drs. L. J. Cahill, J. W. Freeman, and A. W. Schardt of the Office of Space Sciences, NASA.

The final report is presented in four separately-bound parts:

Part I - The Energy Spectra of Electrons from Pion-Muon-Electron Decays in Interstellar Space;

Part II - High Energy Gamma Rays from Cosmic Ray Collisions in Space;

Part III - Low Energy Protons from Cosmic Ray Collisions in Space;

Part IV - Cosmic Ray Hazards in the Solar System.

SUMMARY

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A major source of very high energy cosmic gamma rays is the collision of high energy cosmic ray protons with intergalactic gas. High energy gamma rays are produced directly through the decay of neutral pions which are present as secondaries in the collision. The Landau-Milekhin relativistic hydrodynamic model, which visualizes the colliding high energy protons as fluids, is used to obtain the pion distributions in both energy and angle; these are in good agreement with available data. A source function for gamma rays of energy above 10 Bev is found by combining the pion production and decay spectra with the primary cosmic ray proton flux. The resulting gamma ray spectrum follows a different power law than spectra based upon the usual assumption of a line spectrum for the pions in the center of mass system of the colliding protons. The high energy gamma ray intensity in space is calculated for a simple Euclidean expanding universe. By comparison with previous estimates for the proton photoproduction process, it is found that proton-proton and proton-photon collisions appear to contribute about the same order of magnitude to the intergalactic gamma ray intensity above $\sim 10^{15}$ ev. The calculated intensities are well within the observed upper limits.

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1. INTRODUCTION

Cosmic gamma rays with energies above 100 Mev are expected to be produced by the bremsstrahlung of high energy electrons against intergalactic gas, and by the (inverse) Compton scattering of high energy electrons with thermal photons in interstellar and intergalactic space. In addition, the collisions of high energy cosmic ray protons with gas nuclei and photons will produce gamma rays (Ref. 1).

While high energy electrons dissipate energy more rapidly than protons, it has been estimated that gamma ray production by electrons is important in the "low" energy range 100 Mev to 100 Bev. The physics of these electron processes is well understood and the accuracy of the calculations of the gamma flux is limited only by the astrophysical data. However, the physics of high energy proton collisions is not well known, and the calculations to date of the resulting gamma ray flux are based on very simple models.

In the present report, a more exact treatment of the production of gamma rays by proton-proton collisions is given than in earlier simple calculations. The gamma rays result from the prompt decay of neutral pions produced in the collisions. The other significant proton collision process which produces gamma rays is photoproduction; this occurs when protons undergo collisions with thermal photons in space (Ref. 2):

$$\begin{array}{c} \gamma + p \rightarrow \pi^\circ + p \\ \pi^\circ \rightarrow 2\gamma \end{array} \qquad \text{(I)}$$

The nonelectromagnetic process considered here, occurs when high energy protons collide with gas nuclei:

$$\begin{array}{c} p + p \rightarrow p + p + a\pi^+ + b\pi^- + c\pi^\circ \\ \pi^\circ \rightarrow 2\gamma \end{array} \qquad \text{(II)}$$

The relative importance of Eqs. (I) and (II) depends, among other things, on the relative densities of photons and gas in interstellar and intergalactic space. In the present report, it is assumed that production of gamma rays above 10 Bev occurs predominantly in intergalactic space. Recent downward revision of the estimates of the thermal photon density in intergalactic space suggests that the p-p process, Eq. (II), dominates the photoproduction process at lower energies.

2. COSMIC RAYS AND HIGH ENERGY PROTON-PROTON SCATTERING

Experimentally it has been established that the primary cosmic radiation is composed of nucleons with a small fraction of heavier nuclei (Ref. 1). At energies above a few dozen Bev, only the nucleons are important for our present purposes, the alpha particles forming a few per cent and heavier elements less than one per cent of the particle flux. As for the nucleons at the energies under consideration, one expects to find only protons or antiprotons, because the mechanism by which particles are accelerated to high energy is generally assumed to be interaction with stellar, galactic, or intergalactic magnetic fields which would have only a weak effect on neutrons or antineutrons (not zero, however, due to their nonzero quadrupole moments). Experimentally, the antiproton flux is negligible, and the high energy neutron flux is undetected but probably also negligible. Hence, ultrahigh energy protons are the main component of the primary radiation, and in this investigation other particles will be neglected.

Protons may produce electrons and photons directly by bremsstrahlung and pair production in the galactic magnetic fields. However, calculation shows that the resulting flux does not compete with the much larger flux resulting from the production of pions in the collisions of the cosmic protons in space, and the pions' subsequent decay via the mechanisms:

$$\begin{aligned} \pi^{\pm} &\to \mu^{\pm} + \nu_{\mu} \\ & \llcorner\!\!\to e^{\pm} + \nu_{e} + \nu_{\mu} \end{aligned}$$

$$\pi^{0} \to 2\gamma \ .$$

The mesons are produced by the cosmic ray protons colliding with the interstellar medium which consists primarily of low energy protons and photons (starlight) (Ref. 2). In the present report we calculate the meson flux produced by proton-proton collisions only. The pions are produced copiously in the collisions of very energetic nucleons as a result of the strong interaction.

In addition to pion production in collisions of high energy protons, about one-sixth of all the particles produced are K-mesons, with the decay schemes:

$$K^{\pm} \to \mu^{\pm} + \nu_{\mu}$$

or

$$\to \pi^{\pm} + \pi^{0}$$

$$K_1^0 \to \pi^{+} + \pi^{-}$$

or

$$\to 2\pi^{0}$$

$$K_2^0 \to \pi^{\pm} + \mu^{\mp} + \nu_{\mu} \text{ , etc.}$$

There is also a very small production of nucleons and hyperons. The production of these particles may be neglected because they make only a small contribution to the stable end products.

The experimental study of the direct interaction of nucleons encounters several handicaps. First, at ultrahigh energies the primary cosmic ray flux is sufficiently small so as to make events extremely rare, requiring either large sized detectors to be placed at high altitudes or more indirect study of extensive air showers (Ref. 3). Second, when an event is recorded, the extreme energies involved make identification of the primary and secondary particles, and assignments of energies, nearly impossible. Hence the interpretation of one event can vary widely from one investigation to another. Also, the statistical methods applied are usually subject to large error (Refs. 4 and 5) and are based on some dubious assumptions. Finally, experiments usually involve the collision of the proton with some larger nucleus and this may obscure some features of the nucleon-nucleon effects (Ref. 1). Hence, one rarely claims better than order of magnitude accuracy in statements at present, and trends "observed" are often more subjective than objective.

The information available is not very satisfactory from the theoretical standpoint either. The conventional approaches to particle production via field theory, strong coupling theory, dispersion relations, etc., yield impossibly complicated expressions when more than two pions are involved and thus are useless above a few Bev. The nonlinear and nonlocal approaches of Heisenberg (Ref. 6) and Wataghin (Ref. 7) are not yet in a state where calculations can be checked with data to any degree of confidence. Hence, many highly simplified models have been proposed, none of which is very satisfactory in a physical sense, and several of which are hardly distinguished from each other by the data. Most of these models are of a statistical or thermodynamic nature and are summarized nicely in review articles by Koba and Takagi (Ref. 8), Feinberg (Ref. 9), and Kretzschmar (Ref. 10). The important features of these models are outlined briefly below.

i) Fermi Model (Ref. 11): In the center-of-mass system (CMS) two nucleons collide forming a highly excited fluid which is treated thermodynamically. The fluid is characterized by a temperature which is determined by the total CMS energy. The fluid then decays isotropically into particles with a probability dependent only on the statistical weight of their final states. The total number of particles can be made to agree with experiment, but the theory predicts far more K-mesons and nucleons than are actually produced (and with much higher energy). However, the model is still useful at lower energies (see Ref. 12).

ii) Heisenberg Model (Ref. 13): The colliding nucleons are treated hydrodynamically using a shock wave model derived from nonlinear field theory (Ref. 6). This model predicts too high a meson multiplicity in its present form. A pion multiplicity proportional to the CMS energy (square root of the lab energy) is found, whereas experiments indicate that the number appears to be the square root of the CMS energy (fourth root of the lab energy).

iii) Landau Model (Ref. 14): This also is a hydrodynamical model, similar in some respects to Heisenberg's model and in others to Fermi's. When the nucleons collide they form a highly excited fluid volume characterized by the same temperature as in the Fermi model. However, this volume now expands rapidly according to laws of relativistic hydrodynamics. It cools during the expansion, finally decomposing into individual pions at a relatively low temperature corresponding to about the pion rest energy. This model predicts the right number of particles, and a reasonable energy spectrum. The angular distribution can be made to fit the experimental Kaplon-Ritson curves (Ref. 4) of Rozental' and Chernavskii (Ref. 15) by employing the Milekhin (Ref. 16) form of the theory. There has been some field theoretic justification of this model (Ref. 17); it appears to fit the data better than any of the others, and it is in closed form. Therefore, it seems the most fruitful to adopt for further study.

iv) "Two-center" models (Ref. 18): The above models have the general fault that they predict too high an inelasticity in the CMS. These defects can be remedied if we suppose that in the CMS the pions are produced from more than one radiating center (recently events have been found requiring 5 or 6 centers to fit the angular distribution). Kraushaar and Marks (Ref. 18) proposed a two-center excited nucleon model whereby each center is merely an excited state of the nucleon. This has been improved in the "fireball" model (Ref. 18) in which each nucleon leaves behind a slowly moving, highly excited cloud which decays in its own rest frame according to one of the above models. Many variations are possible. However, the simpler Landau model seems to be the most reasonable choice here.

3. THE LANDAU MODEL

As remarked above, the Landau model is the most promising model available. We begin a detailed analysis of its predictions by establishing notation. Let E_p be the total energy of the incoming proton, the target proton is assumed at rest in the lab or star frame of reference and has a rest energy MC^2 ($\sim$ 1 Bev). We define $\gamma_p = \frac{E_p}{MC^2}$ and $\gamma_\pi = \frac{E_\pi}{\mu C^2}$ where μ = pion mass and E_π is the pion total energy. CMS quantities are distinguished by the use of a "c" index, while lab quantities are not labeled. The γ parameters are related to the velocity $v = \beta C$ by $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$ or $\beta = (1 - \frac{1}{\gamma^2})^{\frac{1}{2}}$ and since $\beta \leq 1$, $\gamma \geq 1$. The CMS frame moves with velocity W in the lab, in the direction of the incident nucleon. We define $\Gamma = (1 - \frac{W^2}{C^2})^{-\frac{1}{2}}$, which will be the ratio $\frac{E_p}{MC^2}$ for any particle in the lab which is at rest in the CMS $(\gamma_c = 1)$. By elementary kinematics $\Gamma = \left[\frac{\gamma_p + 1}{2}\right]^{\frac{1}{2}} \approx \sqrt{\frac{\gamma_p}{2}}$. Note that γ_p is roughly the incident energy in Bev. Since the target proton is at rest in the lab, in the CMS it has energy $E_p^c = MC^2\Gamma$, which is also the energy of the incident nucleon in the CMS. We always consider $\gamma_p \gtrsim 50$.

According to many experiments, the high energy cross section of a nucleon is approximately its geometrical cross section, with the pion Compton wavelength for a diameter: $\sigma \cong \pi(\frac{\hbar}{2\mu C})^2$. We assume that a nucleon at rest is a uniform fluid sphere of this cross section. Therefore, in a system where the nucleons are moving with velocity W, their spheres contract to disks of transverse radius $\frac{\hbar}{2\mu C}$ and thickness $\frac{\hbar}{2\mu C\Gamma} = \Delta$. The time of collision is of order $\frac{\Delta}{C}$ and if a fraction q of the total energy $2E_p^c$ is transferred to the mesons (q is the inelasticity), the <u>action</u> changes by

$$\delta s \approx 2qE_p^c \cdot \frac{\Delta}{C} = \hbar\left[\frac{M}{\mu}\right]q \ .$$

Since

$$\frac{M}{\mu} \approx 7 \ ,$$

$$\delta s \approx 7q\hbar \ .$$

For a classical theory to be valid, $\delta s \gg \hbar$ so we must have $q \sim 1$ ($q \leq 1$ in general). Thus the Landau theory is limited to highly inelastic collisions.

The Landau model (Ref. 14) considers that at the instant of contact of the colliding nucleons ($t = 0$), two shock waves begin to propagate into the disks with velocity C_0^2/C given by relativistic Rankine-Hugoniot relations (Ref. 19). C is the speed of light, and C_0 is the sound velocity in the medium given by thermodynamics (for temperatures $T \gg \mu C^2$, we find $C_0 = \frac{1}{\sqrt{3}} C = 0.557\ C$, while at the lowest temperatures under consideration, $C_0 = 0.538$ C). The colliding matter continues to be compressed, entering the region with velocity $W = (1 - \frac{1}{\Gamma^2})^{\frac{1}{2}} C \approx C$, for 50 Bev incident cosmic rays. At time $t_1 = \frac{C\Delta}{C_0^2 + C^2}$, the shocks reach the outer edges of the disks and matter begins to flow outward from the compacted region of the collision. The region between the outgoing matter front of the shocks and the ingoing matter of the nucleons is the so-called region of the progressive wave solution (Ref. 20), and although it accounts for only one or two pions, these can carry off up to half the available energy. At the time t_1 (Ref. 21) rarefaction waves start to travel back from the outer edges with velocity C_0. They meet at the center at $t_2 = \frac{C}{C_0}(1 + \frac{C}{C_0}) \frac{\Delta}{2C} \frac{C_0^2}{C_0^2 + C^2}$. The entire system at this instant is described by the progressive wave solution. The temperature is still given by $T \cong 2E_p^c$. The system now expands rapidly, pushing the progressive wave regions outward with velocity $\approx$ C. This is the region of the "nontrivial" solution to the hydrodynamic equations (Ref. 21). The expansion, which is largely one dimensional, continues until the temperature approaches $T_c \cong \mu C^2$. Then the complicated three dimensional phase begins (which can only be treated crudely, but governs only the minor transverse phenomena). At $T_c = \mu C^2$ the whole system disintegrates into particles (pions). Actually, many kinds of particles could result. But the probability of production is given (at least for bosons) by $\exp(-\frac{MC^2}{T_c})$, so if $M \gg \mu$, the particle species does not occur significantly. Hence, we get mainly π's and a few K's. The number of particles is proportional to the total change in entropy as T drops from T_0 to T_c. The number of particles produced N_π, or pion multiplicity, is found to be

$$N_\pi = k \left[\frac{E_p}{2MC^2}\right]^{\frac{1}{4}} \cong \frac{k}{2} \left[\frac{E_p^c}{MC^2}\right]^{\frac{1}{2}} \tag{1}$$

where k is given by experimental fit to be $k = 2 \pm 1$. We choose $k = 2$. Agreement is good for E_p from about 50 Bev to 10^4 Bev. Above $E_p = 10^4$ Bev, data are scanty but there is some evidence that N should rise more slowly. Figure 1 (Ref. 22) gives the observed number of charged secondaries produced in proton collisions. Note that the multiplicity varies as $E_p^{\frac{1}{4}}$. The order of magnitude is in fairly close agreement. The angular distribution and energy distribution are described in more detail in Section 4.

Figure 2 shows the stages of development of the nucleons after the collision according to the Landau model. The figure is an adaptation of a figure found in the work of Amai, et al. (Ref. 21).

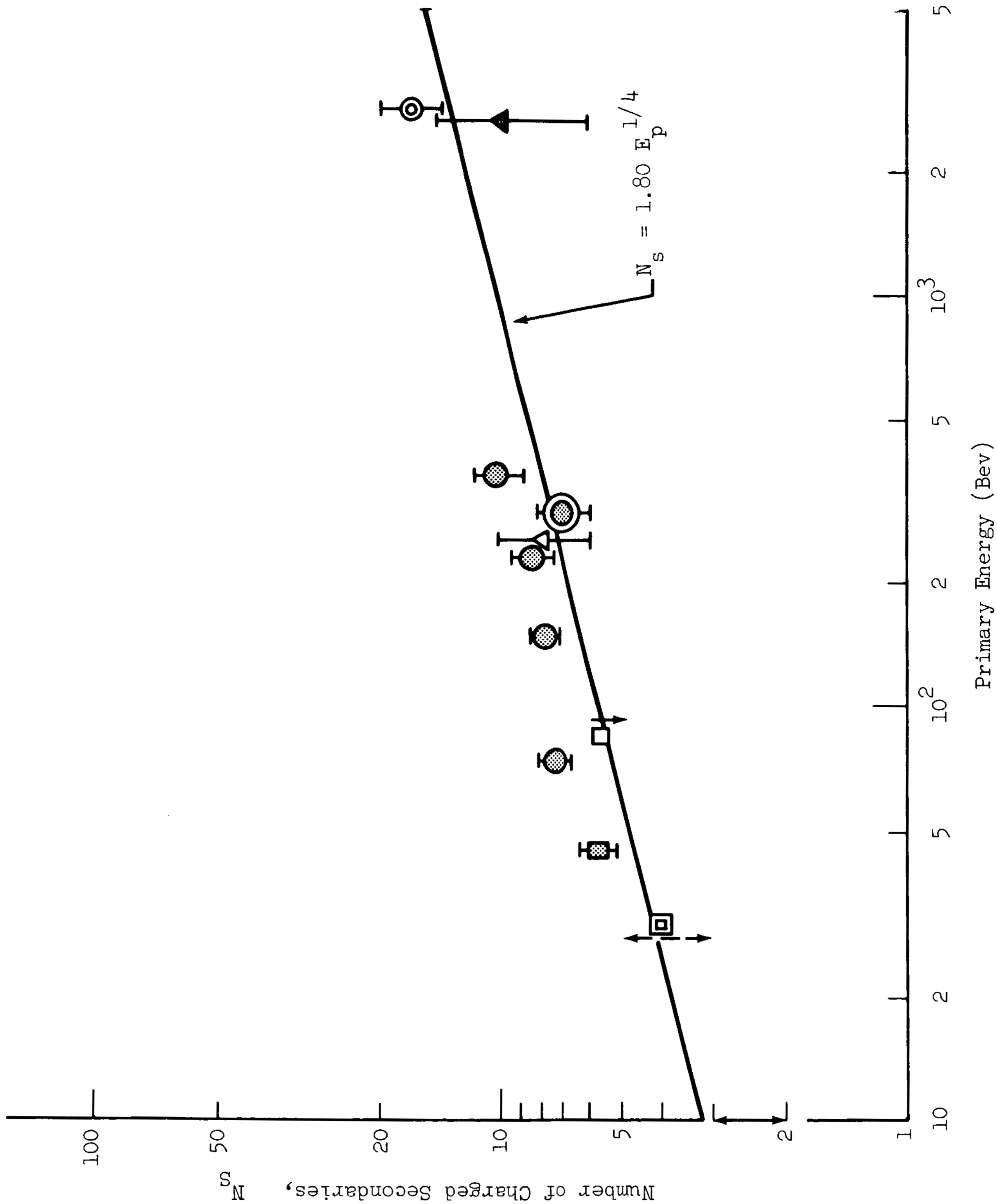
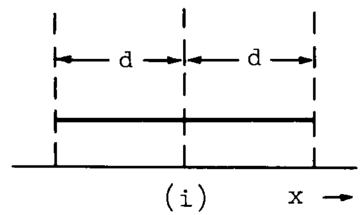
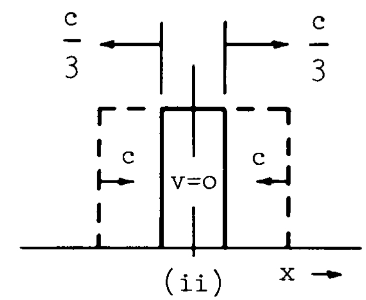


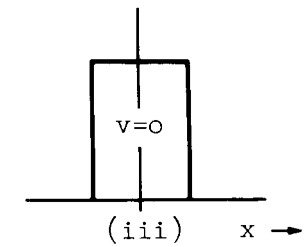
Fig. 1 Charged Particle Multiplicities



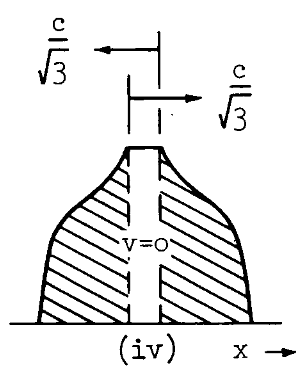
(i) The instant of collision ($t=t_o$)



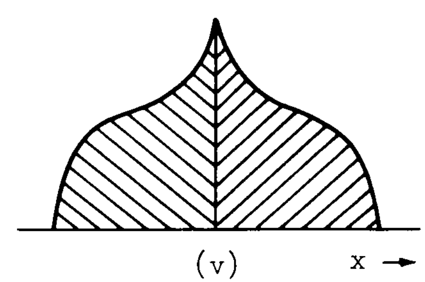
(ii) Shock wave propagates in both directions with velocity $\frac{c}{3}$. The matter between shock continues to be compressed with velocity c; v, the velocity of the fluid, is zero between the shocks



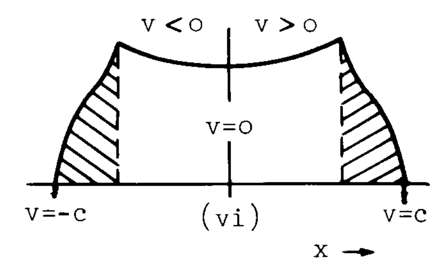
(iii) The shocks have just passed over the matter ($t=t_1$)



(iv) The outflow of matter has begun and its effect (the rarefaction wave) is propagating to the center with sound velocity. The flowing region is represented by the progressive solution (shaded area)



(v) The two progressive (rarefaction) waves have just met



(vi) The region of nontrivial solution bounded by the two progressive waves is formed, and expands quickly, pushing the progressive waves into the narrow space very near the wave fronts

Fig. 2 Nucleon-Nucleon Scattering According to the Landau Method

4. PION ENERGY AND ANGULAR DISTRIBUTION ACCORDING TO THE LANDAU MODEL

a. Center of Momentum Frame of the Colliding Protons

An experimental fact (Ref. 3) of which we must make frequent use is that the average transverse momentum $\overline{P}_{\perp}$ of the pions is relatively independent of E_p, (note that $P_{\perp}$ is a Lorentz invariant $P_{\perp} = P_{\perp}^{c}$) and is measured to be 0.4 - 0.7 $\frac{\text{Bev}}{\text{C}}$. This is the value of $P_{\perp}$, regardless of the angle of emission of the pion, even though the total momentum may be $10^4 \frac{\text{Bev}}{\text{C}}$. In the Landau model, when we completely ignore the three dimensional expansion stage we get the most probable value of $P_{\perp}$, to be 1-2 μC, as a result of thermal motion. This is of the right order. For the overwhelming majority of pions, $P_{\perp} > \mu C$. Figure 3, taken from the work of Milekhin (Ref. 16), shows the corresponding differential transverse momentum spectra for various incident proton energies ($\sinh \zeta \; \alpha \; E_p^{1/14}$). The distribution curve $\frac{1}{N_{\pi}} \frac{dN_{\pi}}{dP_{\perp}}$ versus $P_{\perp}$, as shown in Fig. 3, is peaked near $P_{\perp}/\mu C = 1$ (depending on E_p) with a long tail toward higher $P_{\perp}$.

In the derivation of the energy and angle distribution of the pion in the center of mass of the colliding protons we follow Milekhin (Ref. 16) in defining the parameters η and ζ by:

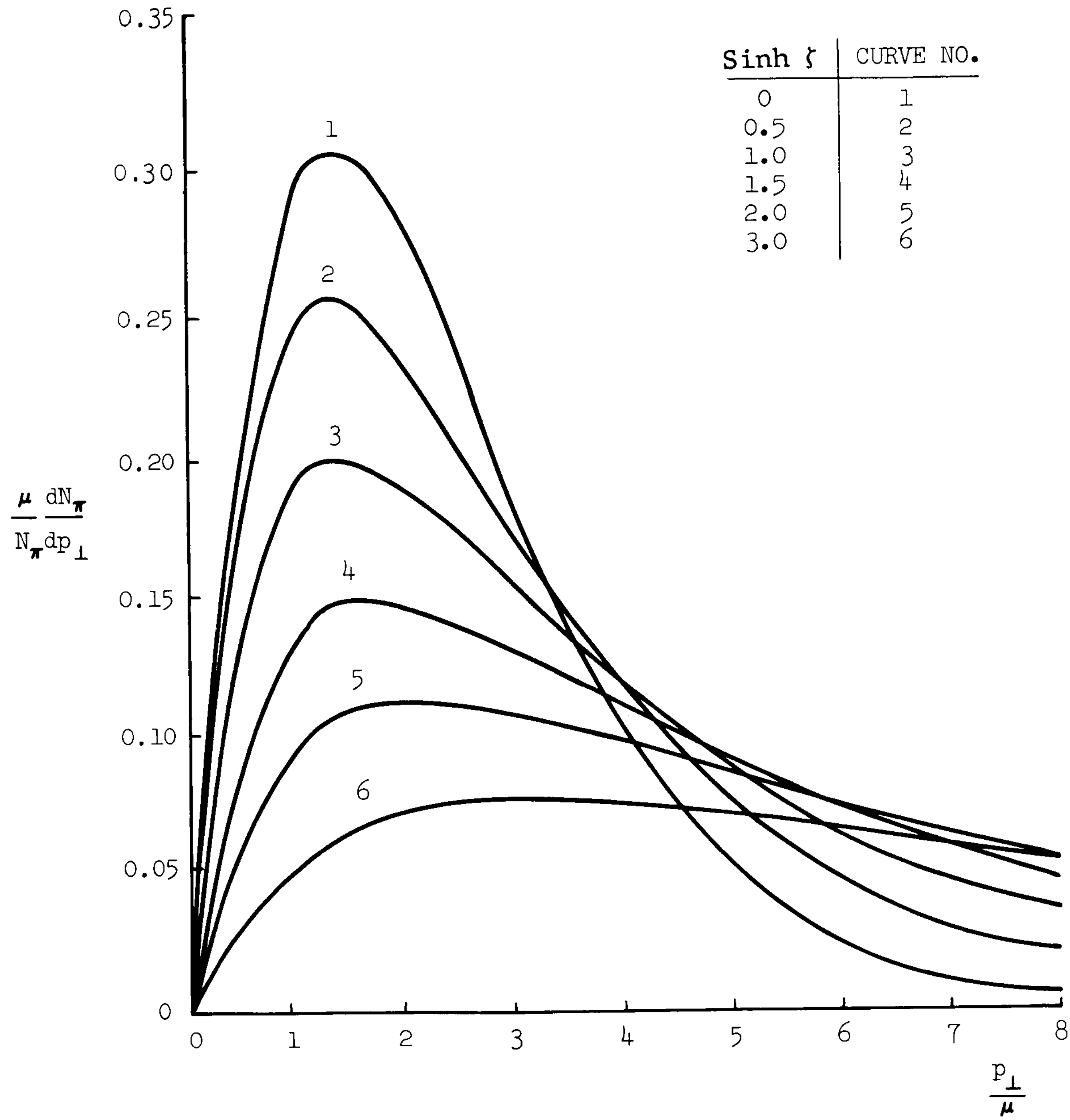


Fig. 3 Pion Transverse Momentum Distribution

$$E_{\pi}^{c} = \mu C^2 \cosh\eta \cosh\zeta \tag{2}$$

$$P_{\perp} = \mu C \sinh\zeta \;. \tag{3}$$

Then the CMS angle is given by

$$\tan\theta_c = \tanh\zeta \operatorname{csch}\eta \;. \tag{4}$$

Milekhin shows that the distribution function

$$\frac{1}{N_{\pi}} \frac{dN_{\pi}}{d\eta} \equiv f_1(\eta) \tag{5}$$

may be approximated to better than 10 per cent accuracy by a Gaussian:

$$f_1(\eta) \cong \frac{1}{\sqrt{2\pi L}} \exp(-\eta^2/2L) \tag{6}$$

the normalization being

$$\int_{-\infty}^{\infty} f_1(\eta)d\eta = 1 \quad . \tag{7}$$

The "Landau parameter" L is chosen by Milekhin (Ref. 16) to be

$$L = 0.56 \,\ell n \frac{E_p}{MC^2} + 1.6 \tag{8}$$

for nucleon-nucleon collisions; this is slightly higher than the conventional value in order to get a better fit to the angular distribution.

To derive the energy and angle distributions we would also need $f_2(\eta, \zeta)$, the distribution in ζ for fixed η. Unfortunately, $f_2(\eta, \zeta)$ is not in closed form. Therefore, employing the fact that $P_\perp > \mu C$ for the vast majority of particles, we choose $\sinh\zeta > 1$ in Eq. (3), obtaining

$$\tanh\zeta = \frac{1}{\sqrt{1 + \frac{1}{\sinh^2\zeta}}} \approx 1 \quad .$$

Consequently,

$$\tan\theta_c \cong \frac{1}{\sinh\eta}$$

then

$$\eta \cong \operatorname{csch}^{-1}\tan\theta_c \quad . \tag{9}$$

Via the identity

$$\operatorname{csch}^{-1} x = \ell n \left[\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}} \right] ,$$

$$\ell n \left[\frac{1}{\tan\theta_c} + \sqrt{1 + \frac{1}{\tan^2\theta_c}} \right] = \ell n \left[\frac{1 + \sec\theta_c}{\tan\theta_c} \right] = \ell n \left[\frac{\cos\theta_c + 1}{\sin\theta_c} \right] ;$$

therefore,

$$\eta = -\ln \tan \frac{\theta_c}{2} \quad .$$

Also,

$$\left|\frac{d\eta}{d\theta_c}\right| = \frac{1}{2 \tan \frac{\theta_c}{2}} \sec^2 \frac{\theta_c}{2} = \frac{1}{\sin \theta_c}$$

so that the CMS angular distribution is given by

$$f(\theta_c) = \frac{1}{N_\pi} \frac{dN_\pi}{d\theta_c} = \frac{f_1\,(\eta(\theta_c))}{\sin \theta_c} = \frac{1}{\sqrt{2\pi L}} \cdot \frac{\exp\left[-\frac{1}{2L} \ln^2 \tan \frac{\theta_c}{2}\right]}{\sin \theta_c} ; \qquad (10)$$

$f(\theta_c)$ is normalized by

$$\int_0^\pi f(\theta_c)d\theta_c = 1 \quad .$$

The form of this curve is shown in Fig. 4.

The CMS energy can be expressed as a simple function of the CMS angle through the use of Eqs. (2) and (3). If we write Eq. (2) as

$$E_\pi^c = \mu C^2 \cosh \eta \sqrt{1 + \sinh^2 \zeta} \quad ,$$

which through Eq. (3) becomes

$$E_\pi^c = C \sqrt{P_\perp^2 + (\mu C)^2} \cosh \eta \quad ,$$

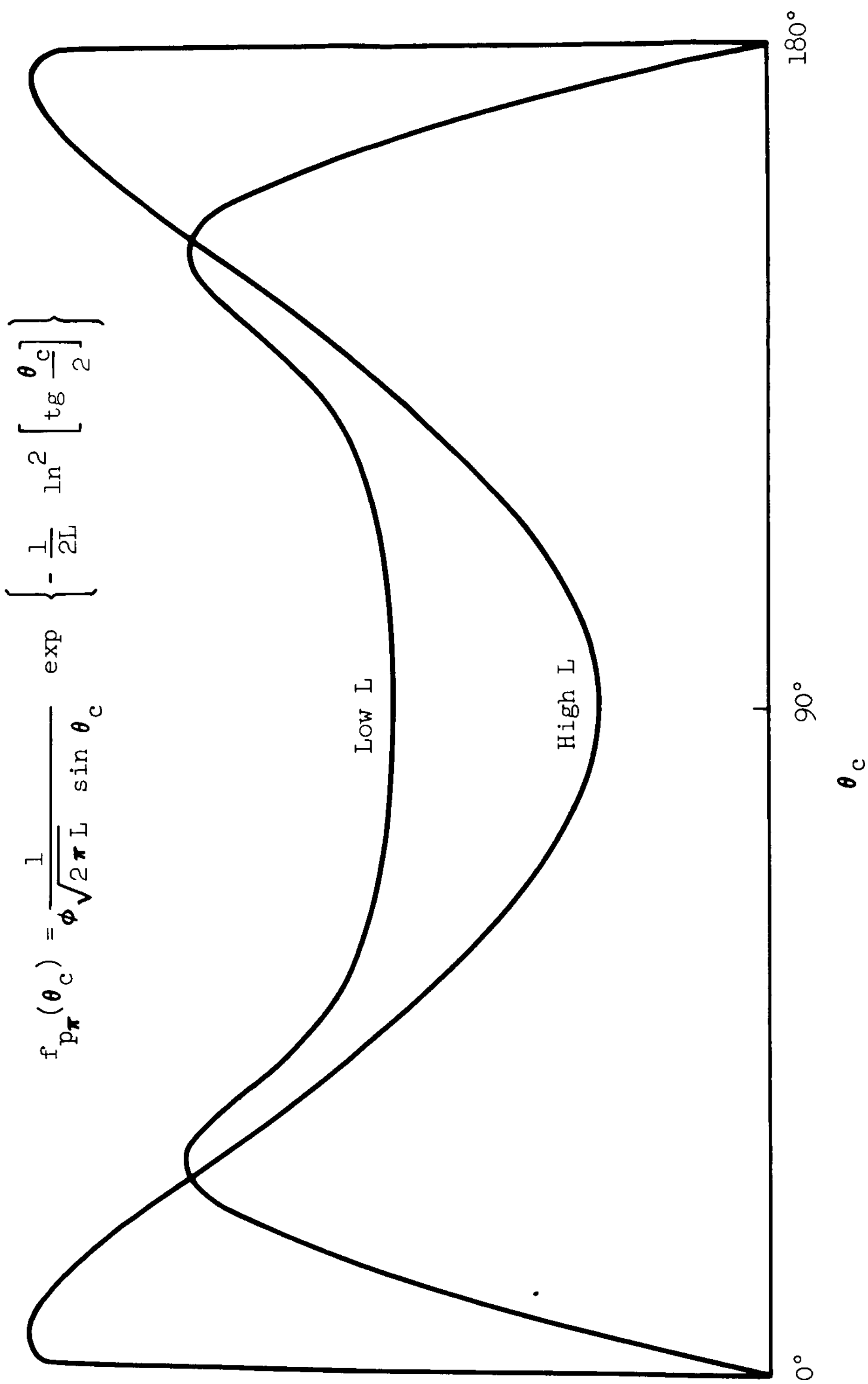


Fig. 4 Center of Mass Angular Distribution of Pions

then through Eq. (9), which gives $\sinh \eta \approx \cot \theta_c$, we may write

$$\cosh \eta \approx \frac{1}{\sin \theta_c}$$

so that

$$E_\pi^c \approx \frac{C\sqrt{P_\perp^2 + (\mu C)^2}}{\sin \theta_c} \quad . \tag{11}$$

Because the transverse momenta of all the pions are of the same order of magnitude, the CMS energy may be approximated by the expression

$$E_\pi^c \approx \frac{C\sqrt{\overline{P_\perp^2} + (\mu C)^2}}{\sin \theta_c}$$

or, by introducing a constant K, which can be determined by experiment, Eq. (11) can be rewritten for all proton energies as

$$E_\pi^c \approx \frac{K}{\sin \theta_c} \quad . \tag{12}$$

One finds $K \sim 2\mu C^2$. (The distortion of the basic Landau-Milekhin theory is minimized by replacing the radical in Eq. (11) by a constant. Figure 3 represents $\frac{dN_\pi}{dP_\perp}$ as peaked in the neighborhood of μC followed by a long tail toward the higher energies. Clearly then, the distribution $\frac{dN_\pi}{d[P_\perp^2]}$ will even be more strongly peaked and consequently the replacement of $P_\perp^2$ by $\overline{P_\perp^2}$ is not a severe distortion of the theory's prediction.)

The divergence in E_π^c at $\theta_c = 0, \pi$ causes no concern because $f(\theta_c) = 0$ there, i.e., there are no particles with $\theta_c = 0, \pi$. Equation (12) allows us to obtain the CMS energy distribution directly from the pions' CMS angular distribution

$$f(E_\pi^c) = \frac{1}{N_\pi} \frac{dN_\pi}{dE_\pi^c} = \frac{1}{N_\pi} \frac{dN_\pi}{d\theta_c} \left|\frac{d\theta_c}{dE_\pi^c}\right|$$

$$= f(\theta_c) \left| \frac{d\theta_c}{dE_\pi^c} \right| .$$

By using Eq. (12) we may readily solve the Jacobian of the transformation

$$\theta_c = \sin^{-1} \frac{K}{E_\pi^c} .$$

Therefore,

$$\frac{d\theta_c}{dE_\pi^c} = - \frac{K/E_\pi^c}{\sqrt{(E_\pi^c)^2 - K^2}} ,$$

but since we may write

$$\tan\frac{\theta_c}{2} = \frac{\sin\theta_c}{1+\cos\theta_c} = \frac{K}{E^c_\pi + \sqrt{(E^c_\pi)^2 - K^2}} ,$$

we find

$$f(E^c_\pi) = \frac{1}{\sqrt{2\pi L}} \frac{1}{\sqrt{(E^c_\pi)^2 - K^2}}$$

$$\times \exp\left\{-\frac{1}{2L}\ell n^2\left[\frac{E^c_\pi + \sqrt{(E^c_\pi)^2 - K^2}}{K}\right]\right\} . \qquad (13)$$

For $E^c_\pi \gg K$, Eq. (13) may be simplified to

$$f(E^c_\pi) = \frac{1}{\sqrt{2\pi L}} \frac{1}{E^c_\pi} \exp\left\{-\frac{1}{2L}\ell n^2\left[\frac{2E^c_\pi}{K}\right]\right\} . \qquad (14)$$

(Equation (14) may be alternatively derived by considering

$$E^c_\pi \cong K \cosh\eta ;$$

for $E^c_\pi \gg K$ we have $\eta \gg 1$, $\cosh\eta \cong \frac{1}{2}e^\eta$, and

$$\ell n\left[\frac{2E^c_\pi}{K}\right] \approx \eta .$$

We rederive Eq. (14) by use of Eq. (6).)

Experimentally Eq. (14) seems to be good from $E_\pi^c \approx K$ to $E_\pi^c \approx \frac{K}{2} e^{\sqrt{2L}}$ for proton energies $E_p \gtrsim 10^3$ Bev. At pion energies above $\frac{K}{2} e^{\sqrt{2L}}$, the experimental spectrum appears to fall drastically. We can revise these distributions to account for this experimental fact: setting $K \approx 2\mu C^2$, we use a distribution of the form given in Eq. (14) for E_π^c between μC^2 and $\mu C^2 e^{\sqrt{2L}}$, and set $f(E_\pi^c) = 0$ outside this region. This requires a renormalization of the distribution to satisfy Eq. (7). Thus, using the "error" integral

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{15}$$

which is tabulated in Jahnke, Emde, and Lösch (Ref. 24) we find

$$f(E_\pi^c) = \frac{1}{\Phi(1)} \sqrt{\frac{2}{\pi L}} \frac{1}{E_\pi^c} e^{-\frac{1}{2L} \ell n^2 \left[\frac{E_\pi^c}{\mu C^2}\right]} \tag{16}$$

for $1 \leq \frac{E_\pi^c}{\mu C^2} \leq \frac{E_{max}^c}{\mu C^2}$, zero otherwise (see Fig. 5).

Here

$$\Phi(1) = 0.84270$$

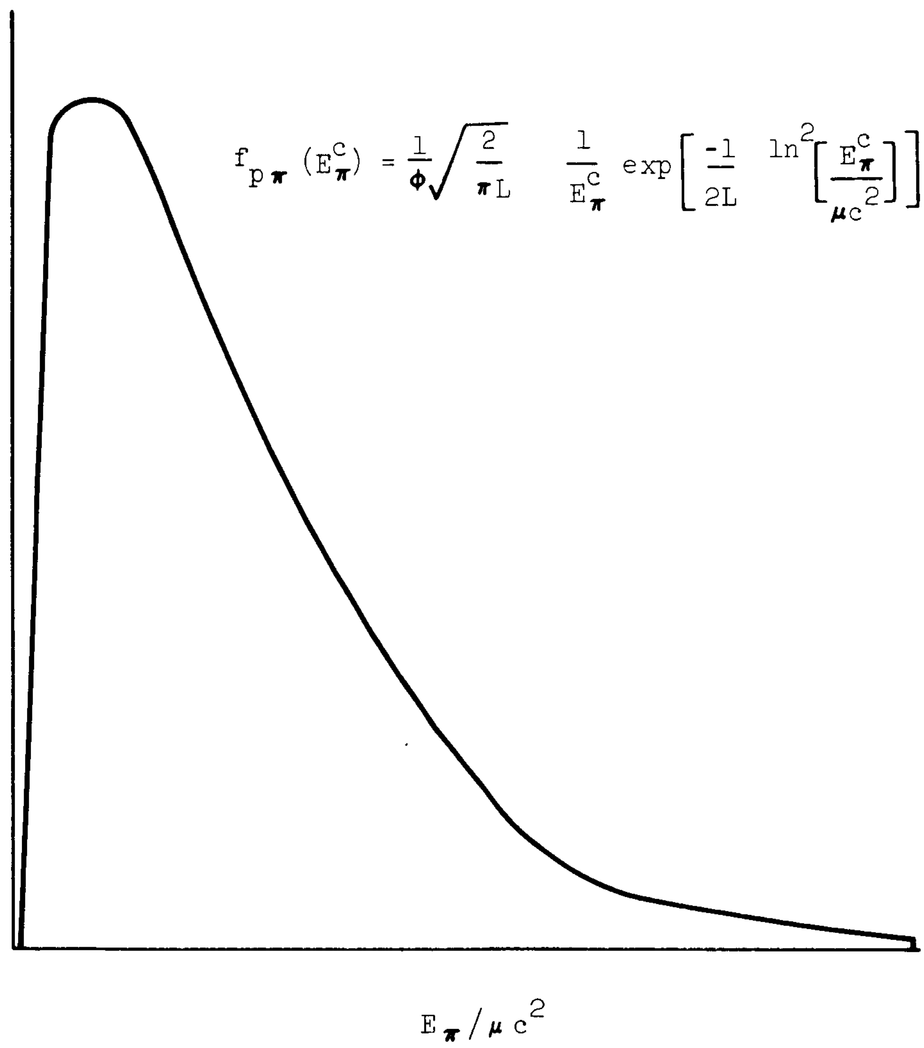


Fig. 5 Center of Mass Energy Distribution of Pions

and

$$E^c_{max} = \mu C^2 \, e^{\sqrt{2L}} = \mu C^2 \, \gamma^c_{max} \quad .$$

$f(E^c_\pi)$ is normalized so that

$$\int_{\mu C^2}^{E^c_{max}} f(E^c_\pi) \, dE^c_\pi = 1 \quad .$$

An even more important reason for truncating the distribution is found by analyzing the mean energy $\overline{E^c_\pi}$ as a function of L (i.e., E_p). If one uses Eq. (14) without a cut off to evaluate $\overline{E^c_\pi}$:

$$\overline{E^c_\pi} = \int_{\mu C^2}^{\infty} E^c_\pi f(E^c_\pi) \, dE^c_\pi = \frac{1}{\sqrt{2\pi L}} \int_{\mu C^2}^{\infty} \exp\left\{- \frac{1}{2L} \ell n^2 \left[\frac{E^c_\pi}{\mu C^2}\right]\right\} dE^c_\pi$$

$$= \frac{\mu C^2}{\sqrt{2\pi L}} \int_0^{\infty} e^{-\frac{x^2}{2L} - x} \, dx \quad \text{by} \quad x = \ell n \left[\frac{E^c_\pi}{\mu C^2}\right]$$

$$= \frac{\mu C^2}{\sqrt{2\pi L}} \, e^{\frac{L}{2}} \int_{-\sqrt{\frac{L}{2}}}^{\infty} e^{-z^2} \, dz \quad \text{by} \quad z = \frac{x}{\sqrt{2L}} - \sqrt{\frac{L}{2}}$$

or

$$\overline{E_\pi^c} = \frac{\mu C^2}{2} e^{\frac{L}{2}} \left[\Phi\left[\sqrt{\frac{L}{2}}\right] + 1 \right] \tag{17}$$

But for large L, $\Phi\left[\sqrt{\frac{L}{2}}\right] \approx 1$ so $\overline{E_\pi^c}$ grows as $e^{\frac{L}{2}}$; this is ridiculous because a glance at Eq. (14) shows that most particles will have $E_\pi^c < \mu C^2 e^{\sqrt{2L}}$, so we should have $\overline{E_\pi^c} < \mu C^2 e^{\sqrt{2L}}$. On the other hand, with the cut off, Eq. (16) gives

$$\overline{E_\pi^c} = \int_{\mu C^2}^{\mu C^2 e^{\sqrt{2L}}} E_\pi^c f(E_\pi^c)\, dE_\pi^c = \frac{\mu C^2}{\Phi(1)} e^{\frac{L}{2}} \left[\Phi\left[\sqrt{\frac{L}{2}}\right] - \Phi\left[\sqrt{\frac{L}{2}} - 1\right] \right] \tag{18}$$

which differs essentially from the preceding result because of the second term. For $\sqrt{\frac{L}{2}}$ large,

$$\Phi\left[\sqrt{\frac{L}{2}}\right] - \Phi\left[\sqrt{\frac{L}{2}} - 1\right] \approx \Phi'\left[\sqrt{\frac{L}{2}}\right] = \frac{2}{\sqrt{\pi}} e^{-\frac{L}{2}}$$

which cancels the increasing $e^{\frac{L}{2}}$ in Eq. (17). In fact, using the crude asymptotic formula

$$\Phi(z) \cong 1 - \frac{2}{\sqrt{\pi}} \frac{e^{-z^2}}{2z} \left\{ 1 - \frac{1}{2z^2} \right\}$$

one finds

$$\frac{\overline{E^c_\pi}}{\mu C^2 \, e^{\sqrt{2L}}} \cong \frac{e^{-1}}{\Phi(1)\sqrt{\pi}} \left[\frac{1}{\sqrt{\frac{L}{2}} - 1} \right]$$

which decreases as L increases and is always less than 1.

The cut off in energy implies a cut off in the angular distribution near $\theta_c = 0, \pi$. A renormalization in the calculation similar to that involved in Eq. (16) gives as the angular distribution:

$$f(\theta_c) = \frac{1}{\Phi(1)\sqrt{2\pi L}} \; \frac{\exp\left\{ -\frac{1}{2L} \ln^2 \tan\frac{\theta_c}{2} \right\}}{\sin\theta_c} \tag{19a}$$

for $\theta^c_{min} < \theta_c < \theta^c_{max}$, where

$$\frac{\theta^c_{min}}{2} = \tan^{-1} e^{-\sqrt{2L}} \; , \; \frac{\theta^c_{max}}{2} = \tan^{-1} e^{\sqrt{2L}} = \frac{\pi}{2} - \frac{\theta^c_{min}}{2} \; . \tag{19b}$$

Equation (19a) goes to zero when the condition $\theta^c_{min} < \theta_c < \theta^c_{max}$ is violated.

We may also calculate the integral distributions $G(E^c_\pi)$ = fraction of particles with CMS energy $< E^c_\pi$. Consider

$$G(E_\pi^c) \equiv \int_{\mu c^2}^{E_\pi^c} f(E_\pi^c)\, dE_\pi^c$$

then:

$$G(E_\pi^c) = \frac{1}{\Phi(1)}\, \Phi\left[\frac{1}{\sqrt{2L}}\, \ln \frac{E_\pi^c}{\mu c^2}\right] \tag{20}$$

$$= 0 \quad \text{if} \quad E_\pi^c < \mu c^2$$

$$= 1 \quad \text{if} \quad E_\pi^c > \mu c^2\, e^{\sqrt{2L}} \; .$$

In Table I we have tabulated $G(E_\pi^c)$. Columns 1 and 2 correspond to incident proton energies of 10^2 and 10^8 Bev respectively; Column 3 is introduced to permit the use of the table for calculations of arbitrary incident proton energies

TABLE 1

ENERGY DISTRIBUTION: $G(E_\pi^c)$

$E_\pi^c/\mu_\pi c^2$ ($E_p=10^2$ Bev)	$E_\pi^c/\mu_\pi c^2$ ($E_p=10^8$ Bev)	$\frac{1}{\sqrt{2L}} \ln \frac{E_\pi}{\mu_\pi c^2}$	$G(E_\pi^c)$
1.24	1.44	.075	.10
1.54	2.08	.15	.20
1.73	2.53	.19	.25
3.00	6.69	.395	.50
6.05	22.20	.638	.75
7.39	29.96	.695	.80
11.02	54.60	.828	.90
13.46	81.45	.910	.95

Similarly we can get the integral angular distribution

$$G(\theta_c) = \text{fraction of particles with } \theta < \theta_c = \int_{\theta^c_{min}}^{\theta_c} f(\theta_c)\, d\theta_c$$

and by Eq. (19)

$$G(\theta_c) = \frac{1}{2} + \frac{1}{2\Phi(1)}\, \Phi\left[\frac{1}{\sqrt{2L}}\, \ell n(\tan \frac{\theta_c}{2})\right]$$

$$= 0 \quad \text{if} \quad \theta < \theta^c_{min}$$

$$= 1 \quad \text{if} \quad \theta > \theta^c_{max} \quad .$$

Note that $\Phi(-x) = -\Phi(x)$. This function is tabulated in Table 2.

TABLE 2

ANGULAR DISTRIBUTION: $G(\theta_c)$

θ_c $E_p=10^2$ Bev	θ_c $E_p=10^8$ Bev	$\frac{1}{\sqrt{2L}}\, \ell n \tan \frac{\theta_c}{2}$	$G(\theta_c)$
39°	17°	- 0.39	0.25
90°	90°		.50
114°	129°	.15	.60
152°	167°	.485	.80
164°	177°	.695	.90
169°	178°	.830	.95

b. Lab Frame of Incident Cosmic Ray Proton

The treatment up to this point has been in the CMS. We now consider the lab frame. The basic formula for the angular transformation is

$$\tan \theta = \frac{1}{\Gamma} \frac{\sin \theta_c}{\cos \theta_c + \frac{W}{v_c}} \tag{21}$$

where v_c is the particle velocity in CMS, W is the velocity of CMS in the laboratory and θ the laboratory angle of the emitted pion relative to the incident proton direction. For most particles $v_c \sim C$ and also $W \sim C$, so we can write

$$\tan \theta \approx \frac{1}{\Gamma} \frac{\sin \theta_c}{\cos \theta_c + 1} = \frac{1}{\Gamma} \tan \frac{\theta_c}{2} . \tag{22}$$

The energy transformation into the laboratory frame depends on both E_π^c and θ_c. It is useful if we use the approximation $v_c \sim C$ again; then we may write a Doppler-like formula

$$\frac{E_\pi}{E_\pi^c} \approx \Gamma(1 + \frac{W}{C} \cos \theta_c) \quad . \tag{23}$$

Equation (12) simplifies the transformation if $W \sim C$; then we may write

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$$E_\pi \approx \Gamma(1 + \cos\theta_c)\frac{K}{\sin\theta_c} = \frac{\Gamma K}{\tan\frac{\theta_c}{2}}$$

or by Eq. (22)

$$E_\pi \approx \frac{K}{\tan\theta} \quad . \tag{24}$$

We can also write the relation between the laboratory and CMS energies

$$E_\pi = \Gamma E_\pi^c \left\{ 1 \pm \sqrt{1 - \left[\frac{K}{E_\pi^c}\right]^2} \right\} = \Gamma\left[E_\pi^c \pm \sqrt{(E_\pi^c)^2 - K^2}\right] \tag{25}$$

where the plus sign is for particles moving forward in CMS, and the minus sign for particles moving backward in CMS ($\cos\theta_c < 0$). If $E_\pi^c \gg K$ these reduce to

$$E_\pi \cong 2\Gamma E_\pi^c \qquad \text{(forward in CMS)} \tag{26a}$$

$$E_\pi \cong \frac{(\mu c^2)^2\ \Gamma}{2E_\pi^c} \qquad \text{(backward in CMS)} \tag{26b}$$

or in terms of the γ parameter $(\gamma = E/\mu c^2)$.

$$\gamma_\pi \cong 2\Gamma\gamma_\pi^c \qquad \text{(forward)} \tag{27a}$$

$$\gamma_\pi \cong \frac{\Gamma}{2\gamma_\pi^c} \qquad \text{(backward)} \quad . \tag{27b}$$

These relations may also be derived kinematically.

The angular distribution of pions in the laboratory can be calculated from the angular transformation through the formula

$$f(\theta) = \frac{1}{N_\pi} \frac{dN_\pi}{d\theta} = f\left[\theta_c(\theta)\right] \left|\frac{d\theta_c}{d\theta}\right| \tag{28}$$

Using Eq. (22) the above Jacobian is found to be:

$$\theta_c = 2 \tan^{-1}(\Gamma \tan \theta)$$

so that

$$\frac{d\theta_c}{d\theta} = \frac{2\Gamma}{\Gamma^2 \sin^2\theta + \cos^2\theta} \quad .$$

We may now introduce the identity

$$\sin \theta_c = 2 \sin \frac{\theta_c}{2} \cos \frac{\theta_c}{2}$$

$$= \frac{2 \tan \frac{\theta_c}{2}}{\sec^2 \frac{\theta_c}{2}} = \frac{2 \tan \frac{\theta_c}{2}}{\tan^2 \frac{\theta_c}{2} + 1}$$

which, using Eq. (22) again becomes,

$$\sin \theta_c = \frac{2\Gamma \tan \theta}{\Gamma^2 \tan^2\theta + 1} = \frac{\Gamma \sin 2\theta}{\Gamma^2 \sin^2\theta + \cos^2\theta}$$

or

$$\sin \theta_c = \frac{\sin 2\theta}{2} \frac{d\theta_c}{d\theta} \quad . \tag{29}$$

Incorporating Eqs. (19a) and (29) into the distribution formula we find

$$f(\theta) = \frac{1}{\Phi(1)\sqrt{2\pi L}} \frac{2}{\sin 2\theta} e^{-\frac{1}{2L} \ln^2(\Gamma \tan \theta)}$$

$$\text{for} \quad \theta_{min} < \theta < 0_{max} < \frac{\pi}{2} \tag{30}$$

$$= 0 \qquad \text{for} \quad \theta > \theta_{max} \quad \text{or} \quad \theta < \theta_{min}$$

where

$$\theta_{min} = \tan^{-1} \left[\frac{e^{-\sqrt{2L}}}{\Gamma}\right]$$

$$\theta_{max} = \tan^{-1} \left[\frac{e^{+\sqrt{2L}}}{\Gamma}\right] \quad . \tag{31}$$

Equation (30) indicates an angular distribution in the lab frame which is peaked strongly in the forward direction. The peak will be to the left of $\tan^{-1} \frac{1}{\Gamma}$, due to the $\frac{1}{\sin 2\theta}$ term in the distribution. For 50 Bev cosmic rays where $\tan^{-1} \frac{1}{\Gamma} = 14.8$ degrees, the distribution peaks in the forward direction at $\theta < 0.6$ degrees. See Fig. 6.

A useful quantity, often referred to in the literature, is the angle in the laboratory corresponding to $\theta_c = 90°$, which (Table 2 or by obvious symmetry about $\theta_c = 90°$) contains half the pions. This angle is often denoted $\theta_{\frac{1}{2}}$ and from Eq. (22)

$$\theta_{\frac{1}{2}} = \tan^{-1} \frac{1}{\Gamma} \quad .$$

Experimentally $\theta_{\frac{1}{2}}$ gives a measure of the incident energy E_p via

$$\Gamma = \sqrt{\frac{\gamma_p + 1}{2}}$$

that is,

$$E_p = MC^2\gamma_p = MC^2(2\Gamma^2 - 1) = MC^2(2 \tan^2\theta_{\frac{1}{2}} - 1) \tag{32}$$

One must be careful about using Eq. (32) in the literature, because experimental situations usually deal with nucleon-nucleus collisions (Refs. 4 and 25) where the assumption of CMS symmetry about $\theta_c = 90°$ is rather dubious.

The particle number is a Lorentz invariant, therefore we may find $G(\theta)$, the integral distribution function in the laboratory frame, merely by transforming angles

$$G(\theta) = \frac{1}{2} + \frac{1}{2\Phi(1)} \Phi\left[\frac{1}{\sqrt{2L}} \ell n(\Gamma \tan \theta)\right] .$$

The table of values, Table 2, is then valid in the laboratory frame if the last two column headings are changed to $\frac{1}{\sqrt{2L}} \times$ $\ell n\ (\Gamma \tan \theta)$ and $G(\theta)$ respectively, and if the center of mass angle columns are ignored.

Tables 3 and 4 give the results of calculations using the above formulas, with incident energies ranging from $E_p \sim 10^2$ Bev to 10^{11} Bev, including cut offs, means, and 80 per cent values of the integral spectra. All angles are given in radians and in circular measure.

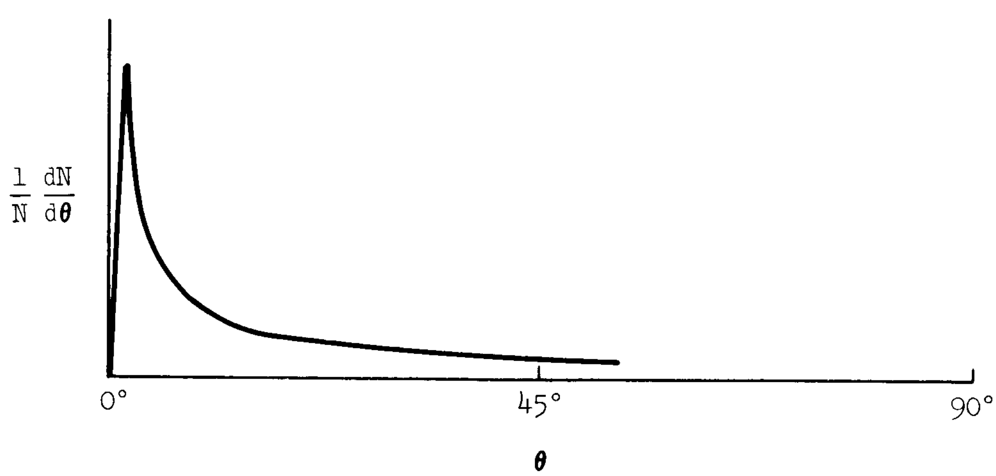


Fig. 6 Laboratory Angular Distribution of Pion Produced in a Single Collision

TABLE 3

ENERGY DISTRIBUTION CALCULATIONS

Incident Energy	Landau Parameter	Expected π Multiplicity	Lab Transf'n of CMS	CMS Energies (π's)		
γ_P	L	N_π	Γ	Max γ_π^c	Mean γ_π^c	80% Limit γ_π^c
10^2	4.18	5.3	7.14	18.0	4.64	7.46
10^3	5.47	9.5	2.24×10^1	27.4	6.11	9.97
10^4	6.76	16.5	7.07×10^1	39.6	8.02	12.8
10^5	8.05	30.0	2.24×10^2	55.1	9.90	16.1
10^6	9.34	53.0	7.07×10^2	75.2	12.5	20.1
10^7	10.63	95.0	2.24×10^3	100.5	15.2	24.5
10^8	11.92	168.0	7.07×10^3	131.6	18.9	29.7
10^9	13.21	299.0	2.24×10^4	170.0	22.1	35.5
10^{10}	14.50	532.0	7.07×10^4	220.0	27.9	42.1
10^{11}	15.79	946.0	2.24×10^5	273.0	33.4	49.4
Ref. Formulas	(8)	(1)		(16)	(17)	Table 1

TABLE 4

ANGULAR DISTRIBUTION CALCULATION

Incident Energy	Landau Parameter	Lab Transf'n of CMS	CMS Extrema Angle ($\theta^c_{max} = \pi - \theta_{min}$)		Lab Angles			
γ_P	L	Γ	θ^c_{min}		θ_{min}	50% Angle Limit	80% Angle Limit	θ_{max}
			(Radians)	(Degrees)	(Radians)	(Radians)	(Radians)	(Radians)
10^2	4.18	7.14	1.12×10^{-2}	6° 22'	7.80×10^{-3}	1.40×10^{-1}	5.17×10^{-1}	1.19
10^3	5.47	2.24×10^{1}	7.30×10^{-2}	4° 10'	1.63×10^{-3}	4.50×10^{-2}	2.18×10^{-1}	8.86×10^{-1}
10^4	6.76	7.07×10^{1}	5.1×10^{-2}	2° 56'	3.61×10^{-4}	1.41×10^{-2}	8.35×10^{-2}	5.06×10^{-1}
10^5	8.05	2.24×10^{2}	3.62×10^{-2}	2° 26'	8.08×10^{-5}	4.47×10^{-3}	3.14×10^{-2}	2.42×10^{-1}
10^6	9.34	7.07×10^{2}	2.66×10^{-2}	1° 32'	1.86×10^{-5}	1.41×10^{-3}	1.14×10^{-2}	1.06×10^{-1}
10^7	10.63	2.24×10^{3}	1.9×10^{-2}	1° 8'	4.44×10^{-6}	4.47×10^{-4}	4.22×10^{-3}	4.50×10^{-2}
10^8	11.92	7.07×10^{3}	1.52×10^{-2}	52'	1.07×10^{-6}	1.41×10^{-4}	1.52×10^{-3}	1.86×10^{-2}
10^9	13.21	2.24×10^{4}	1.16×10^{-2}	40'	2.59×10^{-7}	4.47×10^{-5}	5.40×10^{-4}	7.21×10^{-3}
10^{10}	14.80	7.07×10^{4}	9.0×10^{-3}	31'	6.36×10^{-8}	1.41×10^{-5}	1.92×10^{-4}	3.14×10^{-3}
10^{11}	15.79	2.24×10^{5}	7.2×10^{-3}	25'	1.61×10^{-8}	4.47×10^{-6}	6.88×10^{-5}	1.23×10^{-3}
Ref. Formulas	(8)		(19b)		(31)		Table 2	(31)

The energy spectrum in the laboratory frame may most easily be obtained by returning to the Milekhin form (Ref. 16) of the distribution function

$$f(\eta) = \frac{1}{\sqrt{2\pi L}} e^{-\eta^2/2L} . \tag{6}$$

This technique will give a form for the energy distribution which will be angle independent. A direct relation between E_π and η is obtained by utilizing Eq. (4) and the identity

$$\tan^2 A + 1 = \sec^2 A$$

to get

$$\cos\theta_c = \frac{1}{\pm\sqrt{1+\tan^2\theta}} = \frac{1}{\pm\sqrt{1+\dfrac{\tanh^2\zeta}{\sinh^2\eta}}}$$

so

$$\cos\theta_c = \frac{\sinh\eta}{\sqrt{\sinh^2\eta + \tanh^2\zeta}} . \tag{33}$$

The choice of signs is consistent with Eq. (4) for the physical region

$$0 \leqslant \theta_c \leqslant \pi .$$

We introduce Eq. (33) into Eq. (23) and using Eq. (2) find

$$E_\pi = \mu C^2 \, \Gamma \cosh \eta \cosh \zeta \left[1 + \frac{W}{C} \frac{\sinh \eta}{\sqrt{\sinh^2 \eta + \tanh^2 \zeta}} \right] . \tag{34}$$

Using the standard Landau-Milekhin approximation that

$$\tanh \zeta \sim 1 \quad ,$$

that is that

$$P_\perp > \mu \quad ,$$

we find

$$E_\pi = \mu C^2 \, \Gamma \cosh \eta \cosh \zeta \left[1 + \frac{W}{C} \tanh \eta \right] .$$

Recalling that $\mu C \cosh \zeta = K/C$ in this approximation [see Eqs. (11) through (12)] we find

$$E_\pi = K\Gamma \cosh \eta \, [1 + \frac{W}{C} \tanh \eta] \, .$$

At the lower limit of 50 Bev, the protons will have a center of mass velocity $W = 0.981\ C$, hence $W \approx C$ is an excellent approximation and we may write

$$E_\pi = K\Gamma \, (\cosh \eta + \sinh \eta)$$

$$\therefore \; E_\pi = K\Gamma e^\eta \tag{35}$$

and $$\therefore \frac{dE_\pi}{d\eta} = E_\pi \; . \tag{36}$$

Notice this transformation formula is based only on the experimental observations of $P_\perp$, the conventional approximation of the Landau model, and the assumption that W ($W > 0.98$ C) is of the order of C. The energy distribution in the laboratory frame is obtained immediately from

$$f(E_\pi) = \frac{1}{N_\pi} \frac{dN_\pi}{dE_\pi} = \frac{1}{N_\pi} \frac{dN_\pi}{d\eta} \left| \frac{d\eta}{dE_\pi} \right| \tag{37}$$

as

$$f(E_\pi) = \frac{1}{\sqrt{2\pi L}} \frac{1}{E_\pi} e^{-\frac{1}{2L} \ell n^2 \frac{E_\pi}{K\Gamma}} \tag{38}$$

Equation (38) is normalized upon the assumption of an infinite upper limit for the pion energies. It would be better to normalize the laboratory frame distribution, as had been done for the center of mass distribution [see Eq. (16)] by using an upper pion energy cut off. The center of mass high energy cut off, as can be seen from Eq. (23), becomes in the laboratory frame

$$\begin{aligned} E^L_{max} &= 2\mu C^2 \, \Gamma \, e^{\sqrt{2L}} \\ E^L_{min} &= \frac{\mu C^2}{2} \, \Gamma \, e^{-\sqrt{2L}} \end{aligned} \tag{39}$$

A lower energy cut off must be introduced due to the motion of the center of mass frame in the laboratory frame. Renormalizing requires

$$\int_{E^L_{min}}^{E^L_{max}} f(E_\pi)\, d\, E_\pi = 1 \quad . \tag{40}$$

We find

$$f(E_\pi) = \frac{A}{E_\pi} \quad \exp\left\{- \frac{1}{2L} \quad \ell n^2 \frac{E_\pi}{K\Gamma}\right\} \tag{41}$$

where

$$A = \sqrt{\frac{2}{\pi L}} \left\{ \frac{1}{\Phi(1) + \Phi(t)} \right\} \doteq \frac{1}{\Phi(1) \sqrt{2\pi L}}$$

because

$$1 < t = 1 + \frac{\ell n 4}{\sqrt{2L}} \leq 1.52 \quad \text{for} \quad E_p \geq 50 \text{ Bev}$$

which implies

$$0.8427 = \Phi(1) < \Phi(t) < \Phi(1.52) = 0.9684 \quad .$$

Equation (41) gives the omnidirectional energy distribution function for pions produced in high energy proton-proton collisions; it is illustrated in Fig. 7. This function is based on the

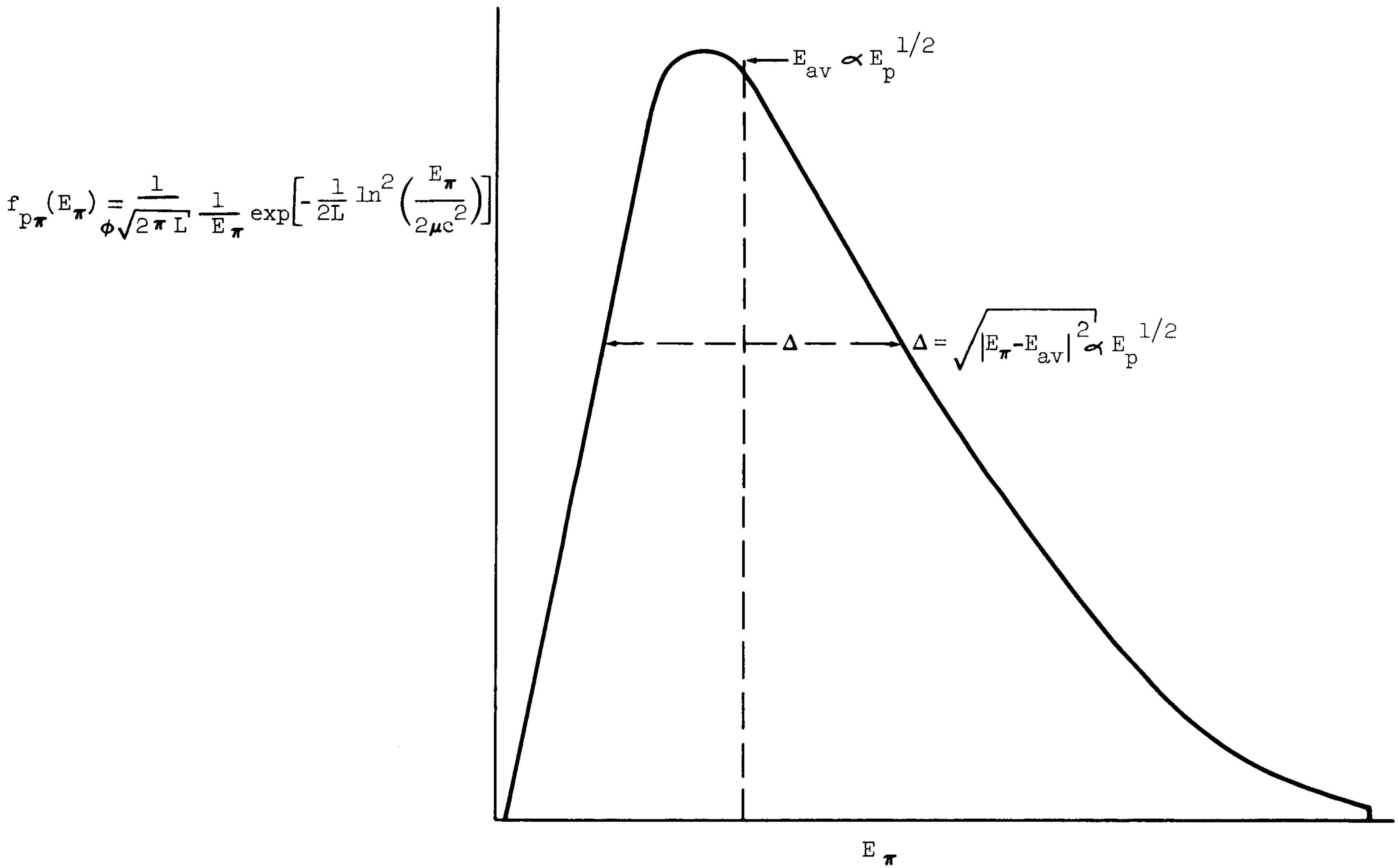


Fig. 7 Laboratory Energy Distribution of Pions

Landau-Milekhin assumptions and the approximation that $W = 1$. This form of the pion distribution will be used extensively in the calculation of the gamma ray spectrum.

5. THE GAMMA RAY PRODUCTION SPECTRUM AT A POINT IN SPACE DUE TO COSMIC RAY COLLISIONS

We now consider the spectrum of gamma rays produced when cosmic rays collide with intergalactic gas. Let $f_{p\pi}(E_p,E_\pi)\,dE_\pi$ be the fraction of pions produced in the range E_π to $E_\pi + dE_\pi$ resulting from collisions of cosmic ray protons (energy E_p) with the interstellar hydrogen, and let $f_{\pi\gamma}(E_\pi,E_\gamma)\,dE_\gamma$ be the fraction of gamma rays with energy in the range E_γ to $E_\gamma + dE_\gamma$ produced from the decay of a pion with energy E_π. Then the fraction of gamma rays in the range dE_γ produced by a collision of a cosmic ray proton of energy E_p is given by

$$f_{p\gamma}(E_p,E_\gamma)\,dE_\gamma = \left[\int dE_\pi f_{p\pi}(E_p,E_\pi)\, f_{\pi\gamma}(E_\pi,E_\gamma)\right] dE_\gamma \;. \tag{42}$$

The integration extends over all pion energies (for a given cosmic ray energy E_p) which could contribute a gamma with energy E_γ. The number of gammas produced per unit volume and time by the collisions of cosmic rays in the energy range E_p to $E_p + dE_p$ with the interstellar gas will be given by

$$N_\gamma(E_\gamma,E_p)\,dE_p = j_p(E_p)\,n(\vec{r})\,\sigma^t_{pp}(E_p)\,m_\pi(E_p)\,m_\gamma dE_p \tag{43}$$

where

$j_p(E_p)$ = the cosmic ray proton flux,

$n(\vec{r})$ = the density of the intergalactic gas,

$\sigma^t_{pp}(E_p)$ = the total cross section for production of pions in proton-proton collisions,

$m_\pi(E_p)$ = the multiplicity of neutral pions per proton-proton collision, and

m_γ = the multiplicity of gammas produced in π° decay = 2.

Combining the number of gammas resulting from cosmic rays of energy E_p with the probability of producing gamma rays with energy E_γ yields the number of gamma rays in the range E_γ to $E_\gamma + dE_\gamma$ produced by cosmic ray protons in the energy range E_p to $E_p + dE_p$:

$$N_{p\gamma}(E_\gamma,E_p)\,dE_p\,dE_\gamma = N_\gamma(E_\gamma,E_p)\,dE_p\,f_{p\pi\gamma}(E_p,E_\gamma)\,dE_\gamma \; . \tag{44}$$

Integration over the spectrum of incident cosmic ray energies then gives the total number of gamma rays produced per unit time and volume in the energy range E_γ to $E_\gamma + dE_\gamma$ resulting from cosmic ray proton-proton collisions in space.

$$q_\gamma(E_\gamma)\,dE_\gamma = \left[\left|\int N_{p\gamma}(E_p,E_\gamma)\,dE_p\right|dE_\gamma\right.$$

$$= \left[n(\vec{r}) \int dE_p\, j_p(E_p)\, \sigma^t_{pp}(E_p)\, m_\pi(E_p)\, m_\gamma\right. \tag{45}$$

$$\times \left. \int dE_\pi\, f_{p\pi}(E_p,E_\pi)\, f_{\pi\gamma}(E_\pi,E_\gamma)\right] dE_\gamma \ .$$

The first step in evaluating Eq. (45) is to determine the distribution function for gamma rays produced when a neutral pion decays. Subsequently, we shall evaluate $f_{p\pi\gamma}(E_p,E_\gamma)$ in Section 5b and $q_\gamma(E_\gamma)$ in Section 5c.

a. <u>The Gamma Ray Spectrum from a Decaying Pion</u>

First consider the gamma ray spectrum from the decaying pions in the rest frame of the pion. Assuming an isotropic distribution of the gamma rays, we find the energy of a gamma ray in the rest frame of the pion as

$$E^*_\gamma = \frac{\mu c^2}{2} \ .$$

If we write the pion's energy in the lab frame as

$$E_\pi = \mu c^2 \gamma_\pi$$

we can write the gamma rays' energy in the lab frame as

$$E_\gamma = \gamma_\pi \frac{\mu c^2}{2}(1 + \beta_\pi \cos \alpha) \tag{46}$$

$$= \frac{E_\pi}{2}(1 + \beta_\pi \cos \alpha)$$

where α is the gamma ray's production angle relative to the pion's line of flight. The distribution of photons in the lab frame will then be as shown in Fig. 8

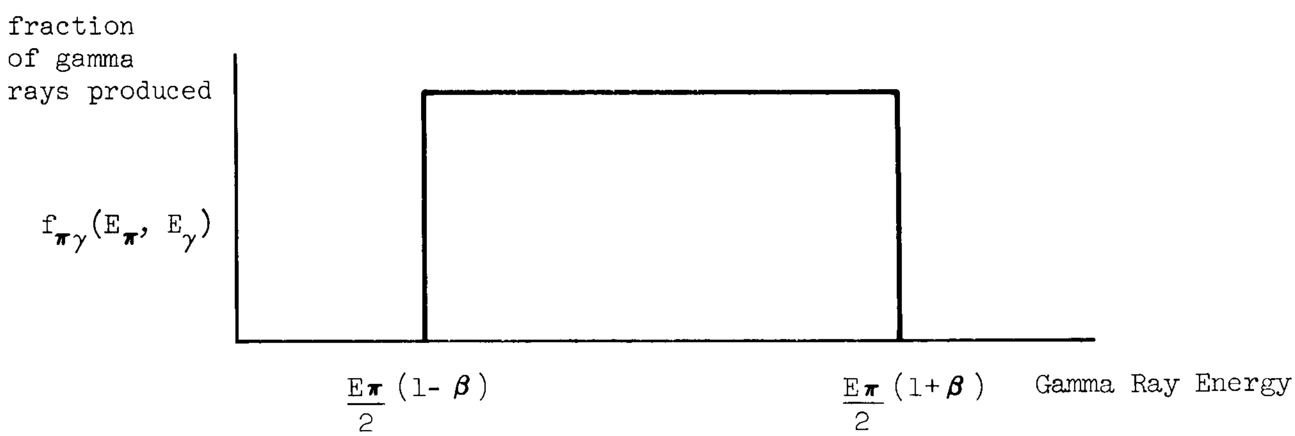


Fig. 8 Laboratory Energy Distribution of Gamma Rays Produced in π° Decay

where the extent of the distribution is given by

$$E_\gamma^{max} = \frac{E_\pi}{2}(1 + \beta_\pi)$$

$$E_\gamma^{min} = \frac{E_\pi}{2}(1 - \beta_\pi) \tag{47}$$

The width of the spectrum will therefore be $E_\gamma^{max} - E_\gamma^{min}$ which we write as

$$\omega = \mu C^2 \gamma_\pi \beta_\pi \equiv p_\pi C$$

where p_π is the magnitude of the "3" momentum of the decaying pion. Employing the normalization required by Eq. (45), we find immediately for this step pulse type distribution that if

$$\int f_{\pi\gamma} dE_\gamma = 1$$

then

$$f_{\pi\gamma} = \frac{1}{p_\pi C} \quad . \tag{48}$$

This is a distribution which is independent of the gamma rays' momentum or energy for a given pion.

b. <u>The Gamma Ray Production Spectrum from a Single p-p Collision of Energy E_p</u>

After the determination of the gamma ray production spectrum from the decay of the π°, the next step in the determination of the complete gamma ray production spectrum is to determine $f_{p\pi\gamma}$, the fraction of gamma rays in a given energy range produced in a single cosmic ray collision. $f_{p\pi\gamma}$ is specified in Eq. (42).

The upper limits in the pion integration in Eq. (42) are determined by the model of pion production, because any pion of energy greater than a given minimum energy can decay into a given gamma ray energy range. Consequently, the pion upper energy limit will only be restricted by the pion production spectrum [Eq. (39)].

The pion lower energy limit is the smallest energy that a pion can have and still decay into the gamma ray with energy E_γ. This minimal energy pion can be found by recognizing that such a pion must have a decay spectrum in which the gamma ray, with energy E_γ, will be the most energetic gamma ray that can be emitted. Therefore, to determine the energy of this pion let us return to the π° decay spectrum calculation and recall Eq. (47) which specified the most energetic gamma ray produced for a given pion and invert this formula to determine the minimal energy pion. Consider then

$$E_\gamma^{max} = \frac{E_\pi}{2}[1 + \beta_\pi] \tag{47}$$

which we now simply specify as

$$E_\gamma = \frac{E_\pi^0}{2}[1 + \beta_\pi^0] \tag{47a}$$

where

$$E_\pi^0 = \text{the minimal pion energy.}$$

Equation (47a) may be written as

$$E_\gamma = \tfrac{1}{2}[E_\pi^0 + p_\pi^0]$$

which may be rewritten as

$$[2E_\gamma - E_\pi^0]^2 = E_\pi^{0^2} - (\mu c^2)^2$$

which leads directly into

$$E_\pi^0 = E_\gamma + \frac{(\mu c^2)^2}{4E_\gamma} \quad . \tag{49}$$

Combining the pion laboratory distribution function [see Eq. (41)] with the gamma ray distribution function [see Eq. (48)] we find the gamma ray production function at a fixed cosmic ray energy to be

$$f_{p\pi\gamma}(E_p,E_\gamma) = \int_{E_\gamma + \frac{(\mu c^2)^2}{4E_\gamma}}^{2\mu c^2 \Gamma\, e^{\sqrt{2L}}} dE_\pi \cdot \frac{A}{E_\pi} \exp\left[\frac{-1}{2L} \cdot \ln^2 \frac{E_\pi}{2\mu c^2 \Gamma}\right] \cdot \frac{1}{p_\pi c} \tag{50}$$

$$\equiv A \int_{E_{min}}^{E_{max}} \frac{dE_\pi}{E_\pi^2} \exp\left[- \frac{1}{2L} \ln^2 \frac{E_\pi}{2\mu c^2 \Gamma}\right] \quad , \tag{51}$$

In Eq. (51) we have replaced $p_\pi c$ by E_π. This is consistent with the Landau-Milekhin approximation that

$$P_\perp > \mu C \ .$$

Equation (51) is soluble if we introduce in the integration with fixed E_p the identity

$$\frac{dE_\pi}{E_\pi} = d(\ell n\ E_\pi) = d\left(\ell n\ \frac{E_\pi}{2\mu C^2 \Gamma}\right)$$

defining

$$x = \ell n\ \frac{E_\pi}{2\mu C^2 \Gamma}$$

so that we arrive at

$$f_{p\pi\gamma}(E_p, E_\gamma) = \frac{A}{2\mu C^2 \Gamma} \int_a^b dx\ \exp\left[-\frac{x^2}{2L} - x\right] \ . \tag{52}$$

If we now define

$$y = x\sqrt{\frac{1}{2L}} + \sqrt{\frac{L}{2}}$$

we find

$$f_{p\pi\gamma}(E_p, E_\gamma) = \frac{A}{2\mu C^2 \Gamma} \sqrt{2L}\ e^{L/2} \int_{a'}^{b'} dy\ e^{-y^2}$$

which is

$$= \frac{A}{2\mu C^2 \Gamma} \sqrt{2L}\ e^{L/2}\ \frac{\sqrt{\pi}}{2}\left[\Phi(b') - \Phi(a')\right] \tag{53}$$

where

$$b' = 1 + \sqrt{L/2}$$

$$a' = \frac{1}{\sqrt{2L}}\left[\ell n\left\{E_\gamma + \frac{(\mu C^2)^2}{2E_\gamma}\right\} - \ell n\ 2\mu C^2\Gamma\right] + \sqrt{L/2}\ .$$

Reintroducing A from Eq. (41) and defining

$$\frac{1}{2\varphi} = \frac{1}{\Phi(1) + \Phi(t)}$$

one may write $f_{p\pi\gamma}$ as:

$$f_{p\pi\gamma}(E_p, E_\gamma) = \frac{e^{L/2}}{4\varphi\mu C^2\Gamma}\left\{\Phi(b') - \Phi(a')\right\}\ . \tag{54}$$

c. The Gamma Ray Production Spectrum

Having calculated the fraction of gamma rays produced at a fixed proton energy, we turn to the calculation of the number of gamma rays produced at a given gamma ray energy, at a point in space, due to scattering of the actual spectrum of cosmic ray protons. The gamma ray production spectrum can be evaluated in the form of Eq. (45)

$$q_\gamma(E_\gamma) = n(\vec{r}) \int dE_p j_p(E_p)\, \sigma^t_{pp}(E_p)\, m_\pi(E_p)\, m_\gamma \int dE_\pi f_{p\pi}(E_p, E_\pi)\, f_{\pi\gamma}(E_\pi, E_\gamma). \tag{45}$$

The inelastic proton-proton cross section for energies greater than 1.5 Bev (10^9 ev) remains constant at 27 mb (mb = 10^{-27} cm^2) (Refs. 26 through 29). The gamma ray multiplicity is of course 2. The primary cosmic ray energy spectrum has been discussed by many authors (Refs. 31 through 34). The other terms in Eq. (45) are given by Eqs. (1), (41), (48), and (54).

In order to accomplish the indicated calculations, let us introduce the transformations

$$\sqrt{\frac{L}{2}} = u \tag{55}$$

and then write for Eq. (53):

$$b' = 1 + u \quad . \tag{56a}$$

Unfortunately, the lower limit a' cannot be easily transformed. Recall:

$$a' = \frac{1}{\sqrt{2L}} \ell n \frac{E_\gamma + \frac{\mu^2 C^4}{4E_\gamma}}{2\mu\Gamma C^2} + \sqrt{\frac{L}{2}} \quad .$$

If we denote the quantity

$$E_\gamma + \frac{\mu^2 C^4}{4E_\gamma}$$

by ε_γ, clearly, ε_γ is approximately equal to E_γ for the range of gamma rays under consideration. We can put a' into the form

$$a' = \frac{1}{\sqrt{2L}} \ln \frac{\mathcal{E}_\gamma}{2\mu\Gamma c^2} + \sqrt{\frac{L}{2}}$$

$$= \frac{1}{\sqrt{2L}}\left[\ln \frac{\mathcal{E}_\gamma}{2\mu c^2} - \ln \Gamma\right] + \sqrt{\frac{L}{2}} \quad .$$

Now modify the empirical definition of the Landau parameter slightly to

$$L = 0.50 \ln \gamma_p + 1.6 \tag{8'}$$

so that

$$\gamma_p = e^{2(L-1.6)} = e^{4u^2-3.2} \tag{57}$$

and approximate Γ by

$$\Gamma \approx \sqrt{\gamma_p/2} \quad . \tag{58}$$

(This approximation is good since the exact expression $\Gamma = \sqrt{\gamma_p/2}\left\{\sqrt{(1 + \frac{1}{\gamma_p})}\right\}$ will be nearly that of Eq. (58) for $\gamma_p \geqq 50$, the region of interest.) We now transform a' through Eqs. (55), (8'), (57), and (58) so that

$$a' = \frac{1}{2u}\left[\ln \frac{\mathcal{E}_\gamma}{\sqrt{2}\,\mu c^2} - 2u^2 + 1.6\right] + u$$

$$= \frac{1}{2u}\left[\ln \frac{\mathcal{E}_\gamma}{\sqrt{2}\mu c^2} + 1.6\right] \quad .$$

Returning to Eq. (54), we write it in "u" representation,

$$f_{p\pi\gamma}(E_\gamma, E_p) = \frac{1}{2\sqrt{2}\,\mu C^2 \varphi}\, e^{-2u^2+1.6}\, e^{u}$$

$$\times \left\{\Phi[1+u] - \Phi\left[\frac{1}{2u}\left\{\ell n \frac{\varepsilon_\gamma}{\sqrt{2}\,\mu C^2} + 1.6\right\}\right]\right\} . \qquad (59)$$

In the calculation of Eq. (45), we shall assume that the cosmic ray flux is functionally the same at all energies in the range $50 < \frac{E_p}{M_p C^2} < \infty$. For example, Morrison (Ref. 35) gives the cosmic ray flux as a power law:

$$j_p = K_p \gamma_p^{-2.5}$$

$$K_p = 0.3 \quad \text{protons} \quad cm^{-2}\ sr^{-1}\ Bev^{-1}\ sec^{-1} \quad .$$

Because there is some uncertainty about the value of the exponent in the power law, we define for use in the analytic part of the calculation

$$j_p = K_p \gamma^{-\eta} \quad . \qquad (60)$$

To calculate the number of gamma rays produced at a point in space

$$q_\gamma(E_\gamma) = n(\vec{r}) \int_{E_p^{min}}^{E_p^{max}} dE_p \cdot \sigma_{pp}^t j_p m_\gamma m_\pi f_{p\pi\gamma}(E_\gamma, E_p) ,$$

the lower bound on the proton's energy is determined by equating the energy of the most energetic pion produced from this lower bound cosmic ray proton to the least energetic pion which is able to produce a gamma ray of energy E_γ. Doing this we write

$$E_{max}^L = 2\mu C^2 \Gamma^{min} e^{\sqrt{2L^{min}}} \tag{39}$$

as the most energetic pion produced in the collision involving the lowest energy proton E_p^{min}, and recall that

$$E_\pi^o = E_\gamma + \frac{(\mu C^2)^2}{4E_\gamma} \equiv \varepsilon_\gamma$$

is the minimum energy a pion must have in order to produce the gamma ray; therefore, we have

$$\varepsilon_\gamma \cong \sqrt{2}\, \mu C^2 (\gamma_p^{min})^{\frac{1}{2}} e^{\sqrt{2L^{min}}} .$$

Thus a lower bound in the u representation, u_{LB}, can be found by setting

$$\varepsilon_\gamma = \sqrt{2}\, \mu C^2 e^{2u_{LB}^2 + 2u_{LB} - 1.6} \tag{61}$$

or

$$u_{LB}^2 + u_{LB} = \tfrac{1}{2}\left\{\ell n\left[\frac{\varepsilon_\gamma}{\sqrt{2}\,\mu C^2}\right] + 1.6\right\} \; . \tag{62}$$

Hence, the lower bound u_{LB} will be given by the solution of

$$u_{LB} = \frac{-1 \pm \sqrt{1 + 2\left[\ell n \dfrac{\varepsilon_\gamma}{\sqrt{2}\,\mu C^2} + 1.6\right]}}{2} \; . \tag{63}$$

We must accept only the positive root since we have defined

$$u \equiv \sqrt{L/2}$$

as a positive number. Consequently,

$$u_{LB} = \frac{-1 + \sqrt{1 + 2\left\{\ell n \dfrac{\varepsilon_\gamma}{\sqrt{2}\,\mu C^2} + 1.6\right\}}}{2} \; . \tag{64}$$

The proton energy corresponding to this limit will be given by

$$E_p^{min} = M_p C^2 \, e^{4u_{LB}^2 - 3.6} \; . \tag{65}$$

We must still be careful with the proton energy range when we fix the gamma ray's energy to prevent contributions from protons of energy below 50 Bev. This is the low bound on the experimental region of validity of the Landau model. Hence, in order not to allow contributions from cosmic ray protons below 50 Bev we must establish a low gamma ray bound, E_γ^{low}, so that all protons considered have energies greater than 50 Bev. This low gamma ray energy can be found by regarding the gamma ray as the most energetic gamma ray released, from the most energetic pion produced, in the collision of a 50 Bev proton with the intergalactic hydrogen. The most energetic pion released by a 50 Bev proton will be

$$E_{max}^{L} = \sqrt{2}\ \mu c^2\ \sqrt{50}\ e^{\sqrt{\ell n\ 50+3.2}}\ .$$

The most energetic gamma ray this pion will release is given by

$$\begin{aligned} E_\gamma &= \frac{E_{max}^{L}}{2}(1 + \beta_\pi^{max}) \\ &\approx E_{max}^{L} \end{aligned} \tag{47a}$$

Consequently,

$$\begin{aligned} E_\gamma^{low} &= \sqrt{2}\ \mu c^2\ \sqrt{50}\ e^{\sqrt{\ell n\ 50+3.2}} \\ &= 19.2 \text{ Bev} . \end{aligned}$$

For astronomical reasons, upper bounds on the cosmic ray energy are very high; however, there is an upper bound on the proton energy due to our imposition of limits on the pion production model [see Eq. (39)]. Therefore, when integrating over proton energies we will arrive at an energy which cannot produce a pion with as small an energy as E_{π}^{0} [see Eq. (49)] for a particular gamma ray energy E_{γ}. This presents a problem in principle but not in practice.

The upper bound on the cosmic ray energy due to the pion production model, can be found by relating ε_{γ} to E_{min}^{L} [see Eq. (39)] and inverting that relationship to solve for E_{p}^{max} (the upper cosmic ray energy) as a function of ε_{γ}. This being accomplished, we arrive at an expression similar to Eq. (64).

$$y = 1 + \sqrt{1 + 2\left(\ell n \frac{2\sqrt{2}\,\varepsilon_{\gamma}}{\mu} + 1.6\right)}$$

where

$$\frac{E_{p}^{max}}{M_{p}C^{2}} = e^{y^{2}-3.2}$$

Since E_{p}^{max} is a monotonic function of ε_{γ} we find that the least value of E_{p}^{max},

$$\bar{E}_{p}^{least} \equiv E_{p}^{max}(\varepsilon_{\gamma}^{low}) \ .$$

Using $\varepsilon_{\gamma} \approx 20$ Bev we find $\bar{E}_{p}^{least} \approx 10^{8}$ Bev. In general, $E_{p}^{max} > \bar{E}_{p}^{least}$. From Eqs. (41) and (48) $\bar{E}_{p}^{least}$ and E_{p}^{min} will produce the same probability of finding a gamma ray [Eq. (39)], but the flux of cosmic rays from $\bar{E}_p^{least}$ is roughly 10^{-20} that of E_p^{min}; consequently, $\bar{E}_p^{least}$ will contribute a negligible fraction to q_γ. Similarly, if we introduce contributions from cosmic ray protons with energies $> \bar{E}_p^{least}$, they too will contribute a negligible fraction to q_γ. Consequently, we may approximate E_p^{max} by ∞ and greatly simplify our calculation.

We now complete the transformation of Eq. (45) into the u representation by writing from Eq. (57)

$$\frac{d\gamma_p}{du} = 8u\ e^{4u^2-3.2}$$

and (66)

$$j_p = K_p\ e^{-\eta(4u^2-3.2)}$$

Using the above transformations and employing the explicit definitions of all the factors Eq. (45) becomes

$$q_\gamma(E_\gamma) = n(\vec{r})\,\sigma_{pp}^{t} \int_{u_{LB}}^{\infty} du \Bigg[8u\ \exp\left\{4u^2-3.2\right\} K_p\ \exp\left\{-\eta(4u^2-3.2)\right\}$$

$$\times\ \frac{2 \times 2}{3 \times 2^{\frac{1}{4}}} \exp\left\{u^2-.8\right\} \frac{1}{2\sqrt{2}\ \mu C^2\varphi} \exp\left\{-2u^2+1.6\right\} e^{u^2} \qquad (67)$$

$$\times\ \left\{\Phi[1+u] - \Phi\left[\frac{1}{u}(u_{LB}^2 + u_{LB})\right]\right\}\Bigg] \quad .$$

Equation (67) is evaluated in Appendix A, where we find

$$q_\gamma = \sigma n(\vec{r})\ \frac{2K}{3}\ \frac{1}{\beta}\left\{\frac{\exp\{-4\beta/4\beta+1\}}{\sqrt{4\beta+1}}\left[1 - \Phi\left[u\sqrt{4\beta+1} + \frac{1}{\sqrt{4\beta+1}}\right]\right]\right.$$

$$+ \frac{\exp\{-4\sqrt{\beta}\ (u^2+u)\}}{2}\left[1 - \Phi\left[u[\sqrt{4\beta} - 1] - 1\right]\right] \tag{68}$$

$$\left. - \frac{\exp\{+4\sqrt{\beta}\ (u^2+u)\}}{2}\left[1 - \Phi\left[u[\sqrt{4\beta} + 1] + 1\right]\right]\right\}$$

where

$$K = n\sigma K_p K_\pi N_\pi^0 e^{5.6}$$

$$\beta = 1.5\ (\eta - 1)$$

Figure 9 indicates that $q_\gamma(E_\gamma)$ may be represented to an excellent approximation by a power law

$$q_\gamma = n\sigma\ \frac{A}{E_\gamma^\zeta} \tag{69}$$

where, if we use $\eta = 2.6$, for the major part of the spectrum,

$$A = 4.09$$

$$\zeta = 3.25\ .$$

Figure 9 compares this result with the Ginzburg-Syrovatskii result (Ref. 1), based on a simple nuclear collision model:

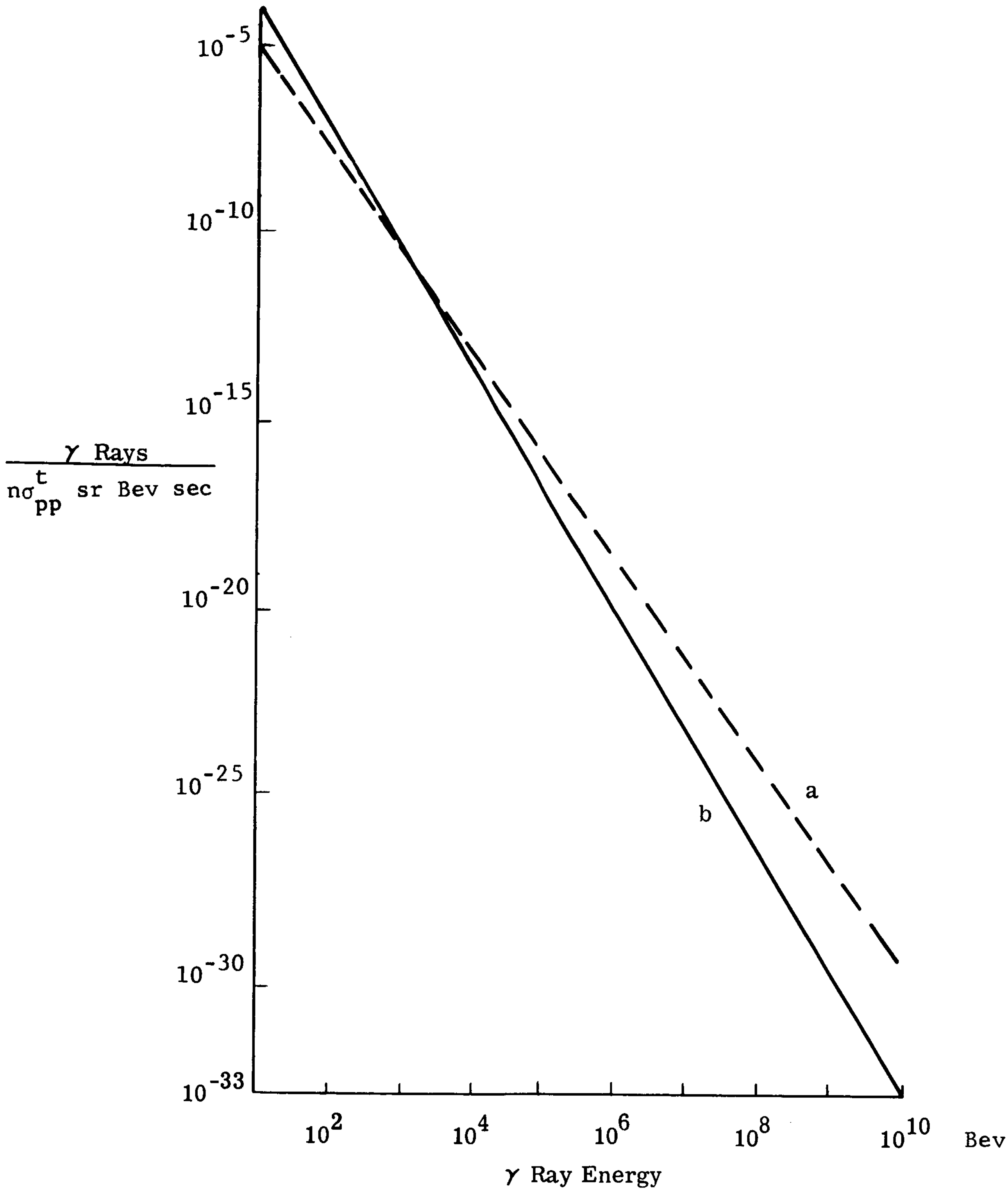


Fig. 9 Gamma Ray Production Spectrum. (a) Ginzburg-Syrovatskii model; (b) Hydrodynamic model

$$q_\gamma^{GS} = n\sigma \times 0.025\ E_\gamma^{-2.8} \tag{70}$$

The two theories give predictions which at $E_\gamma = 1150$ Bev have

$$q_\gamma\ (1150) = q_\gamma^{GS}\ (1150)\ . \tag{71}$$

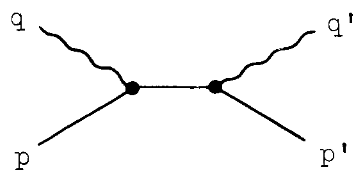
It is important to note that at high energies, of the order of 10^{17} ev, the Ginzburg and Syrovatskii results are 10^3 times larger than our predictions. At low energies our calculation gives results which are about 10 times larger than theirs.

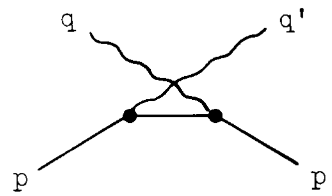
6. ATTENUATION OF GAMMA RAYS BY COLLISIONS WITH MATTER AND RADIATION

Gamma rays are attenuated in their passage through space in two ways: 1) by collisions with local particles and electromagnetic waves in interstellar and intergalactic space (this is discussed in detail in this section); 2) by the effects over large distances of the cosmic red shift (discussed in Section 7).

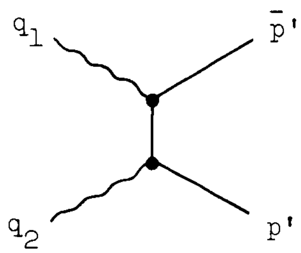
Gamma rays are scattered through electromagnetic interactions. The nature of the electromagnetic interactions incurred by the gamma rays can be put into three major classes. First, there is Compton scattering of the gamma rays by charged particles. (The charged particles in space are predominantly electrons, protons and heavy nuclei.) Secondly, gamma rays are scattered by other photons in space; the result of these interactions may either be the production of particle-antiparticle pairs or the "elastic" scattering of the photons. Lastly, the gamma rays will be scattered by external electromagnetic fields. These three processes are shown schematically in Fig. 10 through the artifice of Feynman diagrams. In the following, we examine these classes and show that only the scattering of gamma rays with photons can be of importance in attenuation in space.

Compton Scattering:

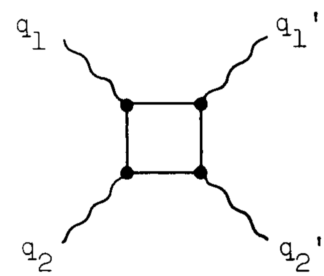




Photon - Photon Scattering

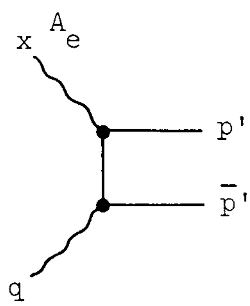


Particle Production

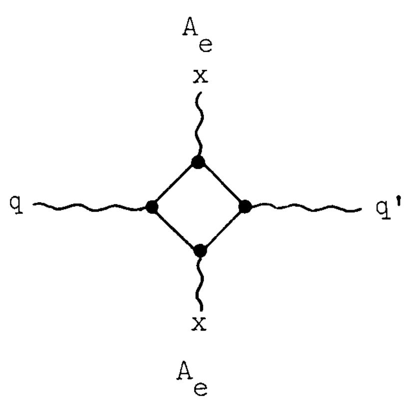


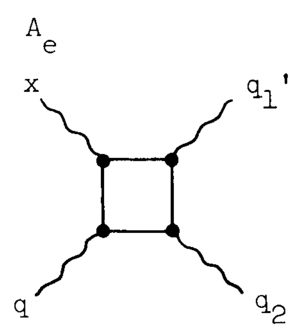
"Elastic"

Scattering by an External Field



Particle Production





Note: The p denotes particles, $\overline{p}$ denotes antiparticles, q denotes photons and A_e external electromagnetic fields. Primes denote appearance of object in final state, unprime indicates initial state object.

Fig. 10 Feynman Diagrams of Electromagnetic Scattering Process

a. Compton Scattering of Gamma Rays by Charged Particles

The basic physics of this process has been examined by many authors (Refs. 36 through 39). Jauch and Rohrlich (Ref. 38) give the total cross section for this process as

$$\sigma_{m_i}(\omega) = 2\pi r_i^2 \left\{ \frac{1+\omega}{\omega^3} \left[\frac{2\omega(1+\omega)}{1+2\omega} - \ln (1+2\omega) \right] + \frac{\ln (1+2\omega)}{2\omega} - \frac{1+3\omega}{(1+2\omega)^2} \right\} \mathrm{cm}^2 \tag{73}$$

where

$$r_i = q_i^2/4\pi m_i$$

q_i = charge of i^{th} particle

m_i = mass of i^{th} particle

and ω, the energy of the gamma ray, in units of the i^{th} particle's rest mass is measured in the rest frame of the i^{th} particle. When $\omega \gg 1$ we can write the extreme relativistic form for Eq. (73)

$$\sigma_{m_i}(E_\gamma) = \pi r_i^2 \frac{m_i}{E_\gamma} \left(\ln \frac{2E_\gamma}{m_i} + \frac{1}{2}\right) \mathrm{cm}^2 \quad \text{(ER)} \quad . \tag{74}$$

Considering gamma rays with energy greater than 20 Bev, Eq. (74)

will predict a total cross section which is a decreasing function of energy for $E_\gamma > 2.72\ m_i$. Table 5 lists the results of employing Eq. (74) for incident gamma rays of 2×10^{10} ev on various particles.

TABLE 5

COMPTON SCATTERING CROSS SECTIONS FOR 20 BEV GAMMA RAYS

Particle	Reduced Energy: ω	Total Cross Section: σ
Electron (Positron)	4×10^4	7.2×10^{-29} cm^2
Muon	200	2.0×10^{-31} cm^2
Proton	20	1.3×10^{-32} cm^2

If we assume some density n_i in space for the charged particles, we can estimate the absorption coefficient $n_i\sigma$ for the gamma rays, and hence the fraction of the gamma rays transmitted. Table 6 presents the absorption coefficient k for gamma rays of energy 2×10^{10} ev and the optical depth τ over a distance $R = 10^{28}$ cm = Hubble radius. The unimportance of attenuation by Compton scattering of gamma rays with energy of 2×10^{10} ev can be clearly seen. Equation (74) shows that gamma rays with energy greater than 2×10^{10} ev will be even less attenuated by Compton scattering.

TABLE 6

COMPTON SCATTERING LOSSES

	n	Reduced Energy ω	k	τ
electrons	10^{-5} cm^{-3}	4×10^{4}	7×10^{-34} cm^{-1}	10^{-5}
protons	10^{-5} cm^{-3}	20	1×10^{-37} cm^{-1}	10^{-9}

b. Scattering of Gamma Rays by Intergalactic Photons

Cosmic gamma rays may scatter with intergalactic photons in either of two ways (as is indicated in the Feynman diagrams of Fig. 10). The elastic scattering process has no threshold, but the elementary particle production process has a threshold energy given through energy conservation. The threshold requirement is

$$E_{\gamma} E_{ph} = m_i^2 \tag{75}$$

where E_{γ} is the energy of the incident gamma ray, E_{ph} is the energy of the thermal photon , and m_i is the mass of the elementary particle or antiparticle. The minimum threshold for particle production is the electron pair production threshold which, for E_{γ} in ev, will require:

$$E_{\gamma} = \frac{0.25 \times 10^{+12}}{E_{ph}} \quad . \tag{76}$$

For values above the pair production threshold, the electron pair production cross section will dominate over the "elastic" scattering cross section because the former is a second order process $(\sigma \propto (\frac{1}{137})^4)$, while the latter is a fourth order process $(\sigma \propto (\frac{1}{137})^8)$. Detailed calculations of these processes can be found in several texts and articles on Quantum Electrodynamics (Refs. 36 through 39).

Jauch and Rohrlich (Ref. 38) report that below the threshold value given by Eq. (76) one finds, to a good approximation, that the elastic scattering total cross section is

$$\sigma = \frac{973}{10,125} \left[\frac{e^2}{4\pi}\right]^2 \frac{1}{\pi^2} \left[\frac{e^2}{4\pi m_e}\right]^2 \left[\frac{E_\gamma \, E_{ph}}{m_e^2}\right]^3 \tag{77}$$

For energies above the threshold, the total "elastic" cross section has not yet been calculated due to the nonelemental form of the differential cross section (Ref. 40). One can make a crude estimate of the total "elastic" cross section by noting that (Ref. 38)

$$\frac{d\sigma}{d\Omega}_{forward} \cong \left[\frac{e^2}{4\pi}\right]^2 \frac{1}{\pi^2} \left[\frac{e^2}{4\pi m_e}\right]^2 \left[\ell n \frac{\sqrt{E_\gamma \, E_{ph}}}{m}\right]^4 \frac{m^2}{E_\gamma \, E_{ph}} \tag{78a}$$

and

$$\frac{d\sigma}{d\Omega}_{backward} \cong \left[\frac{e^2}{4\pi}\right]^2 \frac{1}{\pi^2} \left[\frac{e^2}{4\pi m_e}\right]^2 \frac{m^2}{E_\gamma \, E_{ph}} \quad . \tag{78b}$$

The dominant process above the pair-production threshold, pair production, is calculated in a manner similar to Compton scattering (the pair product; on process is one of the "crossed channels" of Compton scattering). It can be found (Refs. 36 through 39) that the total cross section for pair production is given by

$$\sigma = \frac{1}{2}\left[\frac{e^2}{4\pi m_e}\right]^2 \pi\ (1-\beta^2)\ \left[(3-\beta^4)\ \ln\frac{1+\beta}{1-\beta} - 2\beta(2-\beta^2)\right] \tag{79}$$

where $\beta = \frac{1}{\tilde{E}}\sqrt{\tilde{E}^2 - m_e^2}$; $\tilde{E} = \sqrt{E_\gamma\ E_{ph}}$.

In the nonrelativistic limit of the electron energy $\beta \ll 1$, Eq. (79) may be written as

$$\sigma = \left[\frac{e^2}{4\pi m_e}\right]^2 \pi\beta\ \text{(NR)} \tag{80a}$$

whereas in the extreme relativistic limit of the electron energy, $\beta \rightarrow 1$, Eq. (79) is written as

$$\sigma = \left[\frac{e^2}{4\pi m_e}\right]^2 \pi\ \left[\frac{m_e}{\tilde{E}}\right]^2\left[\ln\frac{2\tilde{E}}{m_e} - 1\right]\ \text{(ER)}\ . \tag{80b}$$

The types of photons which scatter gamma rays can be divided into four classes: 1) radio photons, 2) infrared and ultraviolet photons, 3) thermal photons, and 4) cosmic ray photons. It is probable that the largest contribution to the electromagnetic energy in space comes from the thermal photons (Ref. 41). The radio photons come from the synchrotron radiation of cosmic ray electrons in magnetic fields (Ref. 32); the infrared and ultraviolet photons come from various sources; and the higher energy cosmic ray photons are the subject of this report.

The local energy density of thermal photons appears to be about 0.1 to 0.3 ev cm^{-3} in interstellar space (Refs. 2, 42, and 43) and somewhere in the range 10^{-3} to 10^{-1} ev cm^{-3} in intergalactic space (Refs. 43 and 44). If a solar-like distribution with a temperature kT set at 0.5 ev is taken for the thermal photons (Ref. 43), one finds that the average photon energy will be $\bar{E}_{ph} \approx 1.5$ ev ($\bar{\lambda}_{ph} = 1.86\ \mu$). The number of photons cm^{-3} with energy E_{ph} can be given by the solar type distribution n(E)dE. When the space energy density of thermal photons is taken as ϵ_o ev cm^{-3} one finds

$$n(E)\ dE = \frac{\epsilon_o}{3.29(kT)^3} \quad \frac{E^2}{e^{E/kT} - 1}\ dE\ . \tag{81}$$

Consequently (see Eq. (76)), one would expect predominantly elastic scattering of gamma rays by the thermal photons for gamma ray energies below 1.66×10^{11} ev. Above this energy the gamma rays which interact with photons will be most probably annihilated, forming electron-positron pairs.

An absorption coefficient may be defined for photon scattered gamma rays as

$$k(E_\gamma) = \int_0^\infty n(E)\, \sigma(E_\gamma, E)\, dE \quad , \tag{82}$$

assuming that only a small fraction of gamma rays are scattered into each gamma ray energy region from higher gamma ray energies. In Eq. (82) $\sigma(E_\gamma, E_{ph})$ is the cross section for scattering between an incident gamma of energy E_γ and a thermal photon of energy E_{ph}.

The absorption coefficient due to photon-photon scattering for gamma rays below 1.66×10^{11} ev may be found by introducing Eqs. (77) and (82) into Eq. (81), giving

$$k_{\gamma\gamma}^{e\ell} = \frac{60}{1.645} \left[\frac{\epsilon_o}{m_e}\right] \frac{973}{10{,}125} \left[\frac{e^2}{4\pi}\right]^2 \frac{1}{\pi^2} \left[\frac{e^2}{4\pi m_e}\right]^2 \left[\frac{kT}{m_e}\right]^3 \left[\frac{E_\gamma}{m_e}\right]^3 \tag{83}$$

where $E_\gamma < 1.66 \times 10^{11}$ ev. This clearly is an increasing function with energy; at its upper limit, i.e., 166 Bev, one finds for a temperature of $\frac{1}{2}$ ev that

$$k_{\gamma\gamma}^{e\ell} = 5.48 \times 10^{-31} \left[\frac{\epsilon_o}{m_e}\right] \ \mathrm{cm}^{-1} \quad . \tag{84}$$

For current values of ϵ_o, we may ignore absorption of gamma rays by photons when only elastic scattering is permitted.

The cross section for pair production, as Eq. (80) shows, is peaked in the neighborhood of the threshold energy. This has lead several authors (Refs. 43 and 45) to evaluate the absorption coefficient for gamma rays with energy E_γ against photons in the energy region $E_{ph} = \frac{10^{12}}{E_\gamma}$ ev .

Nikishov (Ref. 43) examines the absorption of gamma rays in the region 1×10^{11} ev to 500×10^{11} ev, and thus has to consider their scattering by thermal photons. Goldreich and Morrison (Ref. 45) examine the absorption of gamma rays in the region of 1×10^{18} to 100×10^{18} ev and hence utilize data on the radiophotons in the metagalaxy. For $R = 10^{28}$ cm, both these studies predict appreciable absorption, as shown in Tables 7 and 8. However, Nikishov has used the high estimate of the intergalactic photon energy density, namely 0.1 ev cm^{-3}; if the low estimate, 10^{-3} ev cm^{-3}, is used, the absorption is very small. For gamma ray energies in the range 10^{16} to 10^{18} ev, observations in the centimeter radio band (Ref. 45) indicate a window. The interesting region between 10^{13} and 10^{16} ev cannot be examined at present because there is a lack of data in the infrared region of the photon spectrum (Ref. 46).

TABLE 7

ABSORPTION COEFFICIENTS FOR PAIR PRODUCTION AFTER NIKISHOV (REF. 43)

10^{-12} E_γ(ev)	0.1	0.5	1	5	10	50
10^{27} $k_{\gamma\gamma}^{ee}$(cm^{-1})	.05	5	7	4	2	.7

TABLE 8

ABSORPTION COEFFICIENTS FOR PAIR PRODUCTION

AFTER GOLDREICH AND MORRISON (REF. 45)

10^{-20} E_γ(ev)	0.01	0.1	0.3	0.5	1.0
10^{+27} $k_{\gamma\gamma}^{ee}$(cm^{-1})	.45	2.5	3.6	4.1	4.7

c. Scattering of Cosmic Gamma Rays by Fields in Space

The physics of the scattering of photons by a static potential has been studied with respect to pair production and Delbrück scattering (elastic scattering); unfortunately all these calculations have been done for the electric field of a nucleus (Refs. 38 and 39). The pair production process has a threshold energy defined by energy conservation to be (Ref. 38)

$$E_\gamma = 2\ m_e \quad . \tag{85}$$

In intergalactic space the dominant electromagnetic field is thought to be the static magnetic field. A magnetic field is described by a vector potential, which invalidates any detailed comparison between the electric field scattering and the magnetic field scattering.

It is possible to set up a quantum electrodynamic calculation with a vector potential which gives finite scattering cross sections. Though a complete calculation of this nature would be interesting, for our purposes it is not needed. An order of magnitude calculation is sufficient to indicate that these external field type interactions can be neglected.

Scattering in an external field will be most interesting in the energy region below pair production in photon-photon scattering. When we compare the magnitude of the lowest order external field - photon scattering cross section, $\sigma_{\gamma B}$, with the elastic photon-photon scattering cross section, $\sigma_{\gamma\gamma}^{\text{elastic}}$ (the competing process in this energy region), we find that

$$\frac{\sigma_{\gamma B}}{\sigma_{\gamma\gamma}^{\text{elastic}}} \sim \frac{\left[\frac{B}{C}\right]^2 \left[\frac{e^2}{4\pi}\right]^2}{\left[\frac{e}{4\pi m_e}\right]^2 \left[\frac{e^2}{4\pi}\right]^2} \sim \frac{\left[\frac{B}{C}\right]^2}{10^{-5}} \quad . \tag{86}$$

The magnetic field is probably of the order of 10^{-7} gauss in intergalactic space and 10^{-6} to 10^{-5} gauss in interstellar space (Ref. 47). At most, the ratio (Eq. 86) would be 10^{-25}, which means that scattering of gamma rays by magnetic fields in space is completely negligible.

Summarizing this section, we see that of all known scattering processes for absorbing gamma rays only photon-photon scattering into electron-positron pairs represents an appreciable absorptive process. This process can only occur for gamma rays with energy that satisfies Eq. (76).

7. HIGH ENERGY GAMMA RAY INTENSITY IN SPACE

To complete our calculations of the high energy cosmic gamma ray intensity, we now combine the production rate of gamma rays with the effects of the cosmic red shift (discussed in Section a) and of attenuation by photon collisions (see Section 6).

At present, it appears likely that high energy cosmic rays are homogeneous and isotropic in interstellar space (Ref. 32). If the small perturbations represented by individual galaxies are ignored, it is also likely that high energy cosmic rays are homogeneous and isotropic in intergalactic space, although there is considerable disagreement about the spectrum and intensity of intergalactic cosmic rays. It is usally considered extremely probable that ultrahigh energy cosmic rays have the same intensity and spectrum inside and outside galaxies. Also, adopting the usual assumption that the bulk of the diffuse matter in the observable universe is intergalactic (rather than in galaxies), it follows that the majority of ultrahigh energy and possibly high energy, proton-proton collisions will occur in intergalactic space. The intergalactic gas density adopted here will be (Ref. 48)

$$n_{IG}(\vec{r}) = 10^{-5}\ \mathrm{cm}^{-3} \quad .$$

a. <u>Combined Red Shift and Attenuation Effects on Gamma Ray Intensity</u>

In order to calculate the combined effect of the absorption and cosmological red shift, consider Fig. 11.

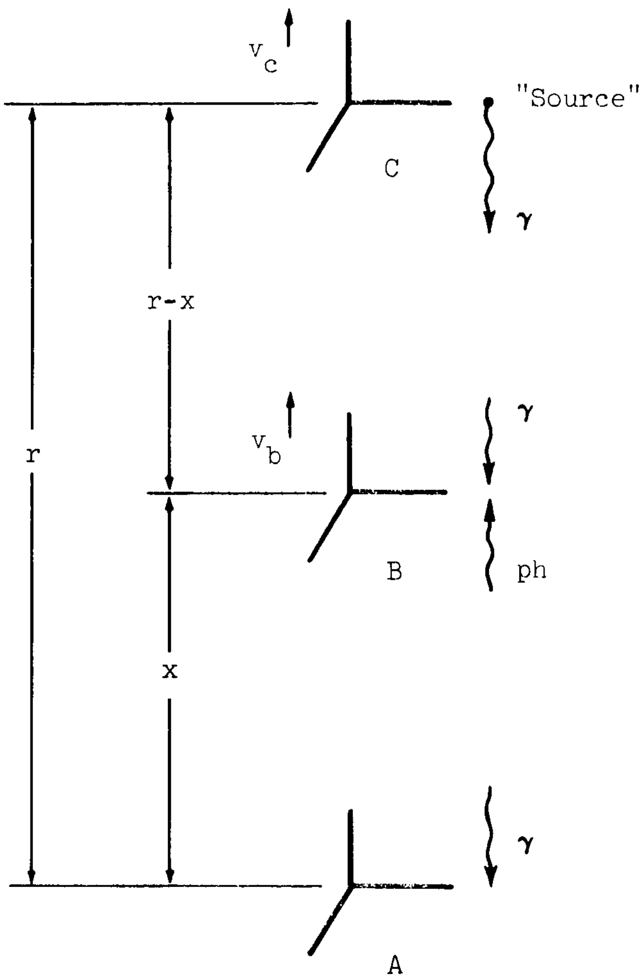
We assume there is a gamma ray "source" at a distance r from an observer such that $rH < \frac{1}{2}$, where H is the "Hubble constant" divided by the speed of light. This condition permits an approximate nonrelativistic calculation for a simple expanding Euclidian universe. In the present treatment, the red shift energy and number effects, but not the relativistic solid angle effects, will be included. The source emits "M" gamma rays per second with energy E_r, which appear to an observer located at A, as source emitting $N(E_o) = \frac{M(E_r)}{1+Hr}$ gamma rays per second with energy E_o.

The presence of scattering requires that the change in the number of gamma rays $\frac{dg}{dx}$ be proportional to the number of gamma rays, g present

$$\frac{dg(x)}{dx} \quad \alpha \quad g(x) \quad . \tag{87}$$

When we consider pair production as the sole absorption phenomenon, Eq. (87) is written as

$$\frac{dg}{dx} = - k_{\gamma\gamma}^{ee}(E_\gamma) \; g \quad . \tag{88}$$



Cosmic Gamma Ray Energy:

$$E_r = E_o\,(1 + Hr)$$

$$E_x = \frac{E_r}{1 + H(r-x)} = E_o\left(\frac{1 + Hr}{1 + H(r-x)}\right)$$

$$E_o = \frac{E_r}{1 + Hr}$$

Fig. 11 Scattering and Relativistic Energy Connections of a Cosmic Gamma Ray

If we wish to calculate the absorption of cosmic gamma rays with energy as measured in our local rest frame A, to be E_o, we must account for the scattering, at location x, of cosmic gamma rays of energy E_x by "ambient" photons such that

$$E_x = E_o \frac{1+Hr}{1+H(r-x)} \quad . \tag{89}$$

If in any frame B (see Fig. 11) we write the distribution of the "ambient" photons, as a function of their energy E_x, $n_x(\epsilon_x)$, then the absorption coefficient as introduced in Eq. (88) will be

$$k_{\gamma\gamma}^{ee}\,\{\omega_r\ x\} = \int_0^\infty d\epsilon_x\, n_x\,\{\epsilon_x\} \cdot \sigma \left\{\omega_o \cdot \frac{1+Hr}{1+H(r-x)}\ ,\ \epsilon_x\right\} . \tag{90}$$

Allowing for scattering only, using Eq. (88), we may write the number of gamma rays transmitted $N(E_o)$, from r to the observer at A, as

$$N(E_o) = M(E_r)\ e^{-\int_0^r k_{\gamma\gamma}^{ee}\,(E_r,\ x)\ dx}$$

$$\ell n\ N(E_o) = \ell n\ M(E_r) - \int_0^r dx \left[\int_0^\infty d\epsilon_x n_x\{\epsilon_x\}\cdot\sigma\{\omega_o\cdot \frac{1+Hr}{1+H(r-x)}\ ,\ \epsilon_x\}\right] .$$

Because the total absorption will be due to the two noninterfering processes, pair production and cosmological red shift effects, we can write the contribution of gamma rays of energy E_o from a source, a distance r from the observer, as

$$N(E_o) = \frac{M(E_r)}{1+Hr}\, e^{-\int_0^r k_{\gamma\gamma}^{ee}(E_r,\, x)\, dx} \tag{91}$$

Using a simple Euclidian model (Ref. 42) for the universe, we can write the cosmic gamma ray intensity $j_\gamma(E_o)$ cm^{-2} sr^{-1} Bev^{-1} sec^{-1} at position A from sources out to a distance of $1/2H$ as

$$j_\gamma(E_o) = \frac{1}{4\pi}\int_0^{\frac{1}{2H}} dr\, \frac{q_\gamma(E_r)}{1+Hr} \cdot \exp\left(-\int_0^r k_{\gamma\gamma}^{ee}(E_o,\, x)\, dx\right) \tag{92}$$

where $q_\gamma(E_r)$ cm^{-3} sr^{-1} Bev^{-1} sec^{-1} is the gamma ray source function given by Eq. (72). Let us write for $q(E_r)$ its power law approximation:

$$q_\gamma(E_r) = AE_r^{-\zeta}$$

where $A = 4$ and $\zeta = 3.20$. To evaluate Eq. (92) we utilize the fact that σ, the total pair production cross section, has a strong maximum in the neighborhood of the threshold energy (see Eq. (75)); hence, it is possible to make the approximation (Refs. 43 and 45)

$$\sigma(E_1,E_2) = \sigma_m \, \delta(E_1,E_2 - m_e^2) \quad .$$

With this approximation, we may write

$$k_{\gamma\gamma}^{ee} \, (E_r, \; x) = n(\epsilon') \; \sigma(E_x, \; \epsilon') \tag{93}$$

where

$$\epsilon' \cong \frac{m_e^2}{E_x} \quad .$$

Furthermore, since σ depends upon the produce $E_x \, \epsilon'$,

$$\sigma(\omega_x, \; E') = \text{constant} \equiv \sigma_m \quad . \tag{94}$$

We assume that in all frames B, all the local observers will see the same distribution of ambient photons within their frame. The energy density of ambient photons in each frame may be written as

$$n_x(\epsilon') = n \left(\frac{m_e^2}{E_o} \; \frac{1+H(r-x)}{1+Hr} \right) \quad . \tag{95}$$

Because we have restricted our region of integration to $r_{limit} = \frac{1}{2H}$, we can see that the argument of the energy density ϵ' will be in the range:

$$\frac{2}{3} \; \frac{m_e^2}{E_o} \leq \epsilon' \leq \frac{m_e^2}{E_o} \quad . \tag{96}$$

We can replace, therefore, ϵ' by some phenomenological radially independent term

$$\epsilon' \sim \eta \frac{m_e^2}{E_o} \tag{97}$$

where $\frac{2}{3} \leq \eta \leq 1$ for the entire region of integration, giving:

$$n_x(\epsilon') \approx n_o \left(\eta \frac{m_e^2}{E_o} \right) \quad . \tag{98}$$

Using Eqs. (94) and (98) we find that $k_{\gamma\gamma}^{ee}$ is independent of radius:

$$k_{\gamma\gamma}^{ee} \approx n_o \left(\eta \frac{m_e^2}{E_o} \right) \sigma_m \quad . \tag{99}$$

b. Calculation of the Gamma Ray Intensity

The integral over x in Eq. (92) can be replaced approximately by $k_{\gamma\gamma}^{ee}$ r, giving

$$j_\gamma(E_o) = \frac{1}{4\pi} \int_0^{\frac{1}{2H}} dr \, \frac{q_\gamma(E_r)}{1+Hr} \, e^{-k_{\gamma\gamma}^{ee} \cdot r} \tag{100}$$

If we express the production spectrum as an explicit function of the radius

$$q_\gamma(E_r) = AE_r^{-\zeta} = AE_o^{-\zeta} \, (1+Hr)^{-\zeta} \quad ,$$

we find that we may factor out the energy dependence from the integral

$$j_\gamma(E_o) = \frac{A}{E_o^{\zeta}} \int_0^{\frac{1}{2H}} dr \, \frac{e^{-k_{\gamma\gamma}^{ee} r}}{(1+Hr)^{\zeta+1}} \quad . \tag{101}$$

This integral is evaluated in Appendix B under the further approximation $\zeta = 3$ finally giving the cosmic gamma ray the intensity as

$$j_\gamma(E_o) = \frac{A}{H} E_o^{-3.2} I \quad . \tag{102}$$

When we wrote

$$\delta = k_{\gamma\gamma}^{ee}/H$$

we have

$$I = e^{\delta}\left\{ \frac{e^{-\delta}}{3}\left[1 - \frac{8}{27}e^{-\delta/2}\right] - \delta\frac{e^{-\delta}}{6}\left[1 - \frac{4}{9}e^{-\delta/2}\right]\right.$$

$$\left. + \delta^2 \frac{e^{-\delta}}{6}\left[1 - \frac{2}{3}e^{-\delta/2}\right] + \frac{\delta^2}{6}\left[(-\mathcal{E}i(-\frac{3\delta}{2})) - (-\mathcal{E}i(-\delta))\right]\right\} .$$

The results of Nikishov (Ref. 43) for energies below 10^{14} ev suggest a value of $k_{\gamma\gamma}^{ee} \sim 10^{-28}$ cm^{-1} which implies a value of $\delta \sim 1$. In this case we find (see Appendix B) that $I = 0.196$ for the energy region $< 10^{14}$ ev. Ginzburg and Syrovatskii's

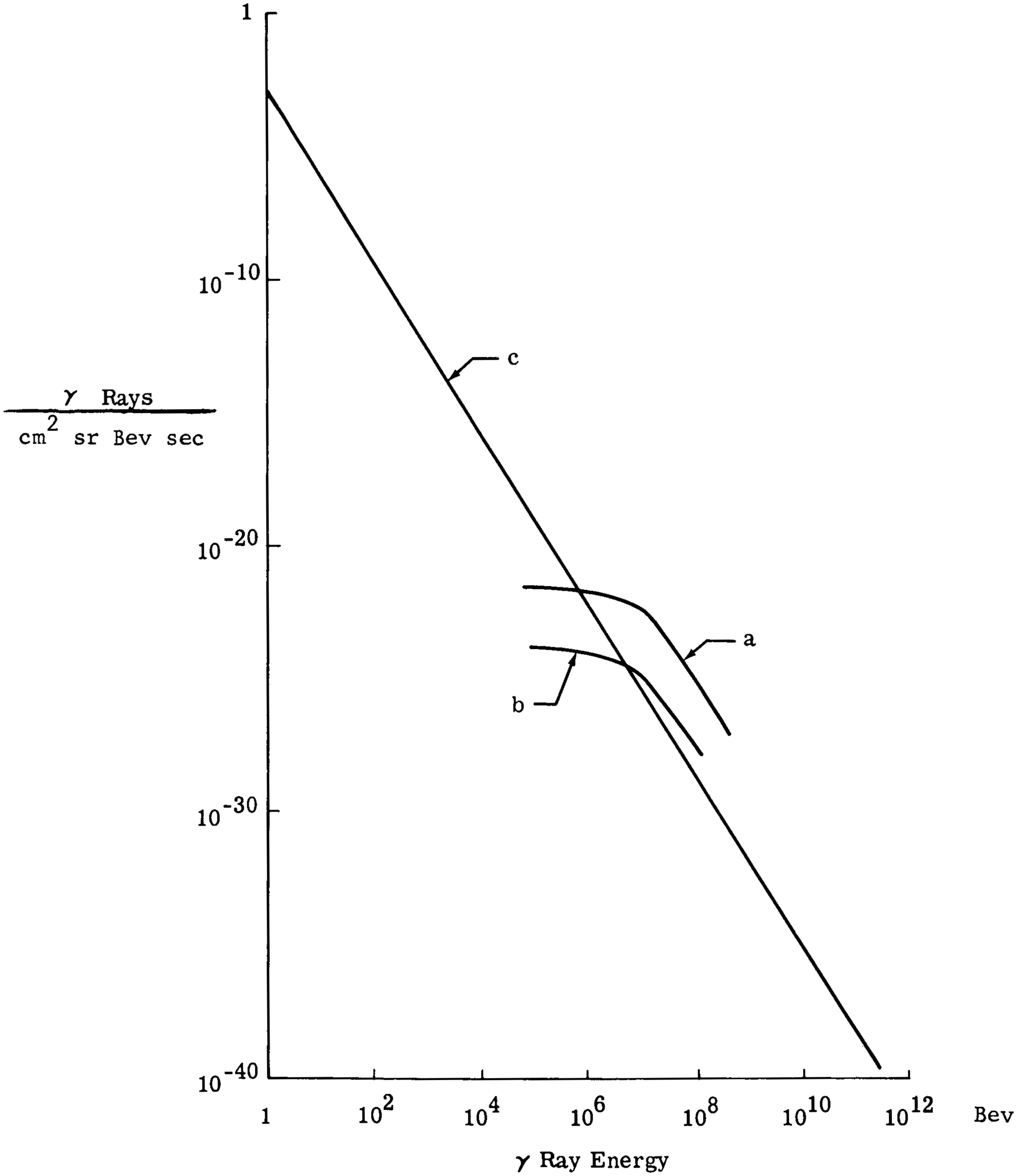


Fig. 12 Gamma Ray Flux in Space. (a) Hayakawa-Yamamoto calculation with n_{ph} = .2 (Ref. 2); (b) Same with n_{ph} = .001; (c) Gamma rays from proton-proton scattering.

suggested value of 10^{-3} ev cm^{-3} (Ref. 44) for the thermal photon density gives $k^{ee}_{\gamma\gamma} \sim 10^{-30}$ ($\delta \sim 0$) so that we find (see Appendix B) I = 0.235 for the energy region $< 10^{14}$ ev. The results of our calculation, using $n_{ph} \sim 10^{-3}$ ev cm^{-3} below 10^{14} ev, and neglecting photon-photon scattering above 10^{18} ev, are compared in Fig. 12 with those of Hayakawa and Yamamoto for photo production (Ref. 2). Their original fluxes, based on a thermal photon density of 0.2 ev cm^{-3}, are presented, as well as the scaled down fluxed based upon a density of 10^{-3} ev cm^{-3}. The low and high energy ends of the spectrum calculated by Hayakawa and Yamamoto depend on the density of hard and soft thermal photons, respectively. While cosmological considerations enter into these photon densities, it is usually assumed that these densities are not particularly high. Thus, it appears that with large uncertainties due to the paucity of information about intergalactic matter and radiation, the proton-proton production process dominates photoproduction below $\sim 10^{15}$ ev, while above 10^{15} ev the contributions of the two processes appear to be of the same order of magnitude.

Various workers have been able to set experimental upper limit estimates on the cosmic gamma ray flux. Chudakov et al. (Ref. 49) and Fruin et al. (Ref. 50) using Cerenkov light detectors have obtained upper limits to the integral gamma ray intensity in the region of 10^{12} ev for various point sources. Unfortunately, without a knowledge of the solid angle observed, we cannot compare our estimates with their values at present. Suga et al. (Ref. 51), using EAS techniques, find an upper limit for the integral flux at 2×10^{15} ev to be 3×10^{-14} cm^{-2} sr^{-1} sec^{-1}, compared with our value of 2×10^{-17} cm^{-2} sr^{-1} sec^{-1} at the same energy. Hence, our theoretical estimate is well within the observational upper limit.

APPENDIX A

EVALUATION OF THE INTEGRAL FORM OF THE GAMMA RAY PRODUCTION SPECTRUM

The gamma ray production spectrum is given by Eq. (67) to be

$$q_\gamma = n\,\sigma \int_u^\infty du \left\{ 8\, e^{4u^2-3.2}\, K_p e^{-\eta[4u^2-3.2]}\, 2 \cdot \frac{K_\pi}{3}\, e^{[u^2-.8]} \right.$$

$$\left. \times\; N_\pi^o\, e^{-2u^2+1.6}\, e^{u^2} \left[\Phi(1+u) - \Phi\left(\frac{1}{u}\,[u_{LB}^2+u_{LB}]\right) \right] \right\} . \qquad \text{(A-1)}$$

In Eq. (A-1) we have replaced the experimental value of the power of the primary cosmic ray spectrum by the parameter "η". In addition we have defined

$$N_\pi^o = \frac{1}{2\sqrt{2}\mu C^2 \varphi}$$

and $\quad K_\pi$ = coefficient of pion multiplicity term.

Let us write Eq. (A-1) as

$$q_\gamma = \frac{2 \times 8}{3}\, \sigma\, K_p\, n\, N_\pi^o\, K_\pi\, e^\alpha \int_{u_{LB}}^\infty u\,du\; e^{-4\beta u^2} \left[\Phi(1+u) - \Phi\left(\frac{1}{u}[u_{LB}^2+u_{LB}]\right) \right] \quad \text{(A-2)}$$

where

$$\beta = \eta - 1$$

and

$$\alpha = 3.2\ \eta - 2.4.$$

In order to evaluate q_γ, it is useful to break Eq. (A-2) into the sum of two integrals such that

$$q_\gamma = q_\gamma^o \times [I_1 - I_2] \tag{A-3}$$

where $$q_\gamma^o = \frac{16}{3}\ \sigma\ K_p\ n\ N_\pi^o\ K_\pi\ e^\alpha$$

$$I_1 = \int_{u_{LB}}^{\infty} udu\ e^{-4\beta u^2}\ \Phi(1+u) \tag{A-4}$$

$$I_2 = \int_{u_{LB}}^{\infty} udu\ e^{-4\beta u^2}\ \Phi[\frac{1}{u}\ (u_{LB}^2+u_{LB})]\ . \tag{A-5}$$

We may then proceed with the evaluation of I_1 and I_2 in turn. The evaluation of I_1 is a straightforward calculation involving integration by parts and the recognition that

$$\frac{d}{du}\ \Phi\ (z) = \frac{z}{\sqrt{\pi}}\ e^{-z^2}\ \frac{dz}{du}\ . \tag{A-6}$$

We may write I_1, using a single integration by parts as

$$I_1 = \frac{-1}{8\beta} e^{-4\beta u} \Phi(1+u) \Bigg|_{u_{LB}}^{\infty} + \frac{1}{4\beta\sqrt{\pi}} \int_{u_{LB}}^{\infty} e^{-(4\beta u^2+u^2+2u+1)} du$$

and finally

$$I_1 = \frac{1}{8\beta} e^{-4\beta u_{LB}} \Phi(1+u_{LB}) + \frac{e^{-4\beta/4\beta+1}}{8\beta\sqrt{4\beta+1}} \left[1 - \Phi\left(u\sqrt{4\beta+1} + \frac{1}{\sqrt{4\beta+1}}\right) \right] . \tag{A-7}$$

The evaluation of I_2 is also through integration by parts, except here one must recognize that

$$\int_a^b \frac{dx}{x^2} e^{-c^2x^2-\frac{f^2}{x^2}} = \frac{\sqrt{\pi}}{4f} \left[e^{2cf} \left\{ \Phi(z_a^+) - \Phi(z_b^+) \right\} - e^{-2cf} \left\{ \Phi(z_a^-) - \Phi(z_b^-) \right\} \right] \tag{A-8}$$

where $z_q^{\pm} = cq \pm \frac{f}{q}$.

Equation (A-8) is discussed in Appendix C. The evaluation of the integral in Eq. (A-8) follows a technique suggested by N. Greenspan of Grumman. A single integration by parts on Eq. (A-5) gives

$$I_2 = \frac{-1}{8\beta} e^{-4\beta u^2} \Phi(1+u)\Bigg|_{u_{LB}}^{\infty} - \frac{u_{LB}^2+u_{LB}}{4\beta\sqrt{\pi}}\int_{u_{LB}}^{\infty} \frac{du}{u^2} \exp\left[-4\beta u^2 - \frac{(u_{LB}^2+u_{LB})^2}{u^2}\right] . \tag{A-9}$$

Using Eq. (A-8) in Eq. (A-9) we get

$$I_2 = \frac{1}{8\beta} e^{-4\beta u^2} \Phi(1+u) + \frac{1}{16\beta}\left\{ e^{4(u^2+u)\sqrt{\beta}}\left[1 - \Phi[u(\sqrt{4\beta}+1)+1]\right]\right.$$

$$\left. - e^{-4(u^2+u)\sqrt{\beta}}\left[1 - \Phi\,[u(\sqrt{4\beta}-1)-1]\right]\right\} . \tag{A-10}$$

Substitution of Eqs. (A-7) and (A-10) into Eq. (A-3) gives

$$N_\gamma = \frac{2}{3} n\,\sigma\,K_p\,N_\pi^o\,K_\pi\,\frac{e^\alpha}{\beta}\left\{\frac{e^{-4\beta/4\beta+1}}{\sqrt{4\beta+1}}\left[1 - \Phi\left(\sqrt{4\beta+1}\;u + \frac{1}{\sqrt{4\beta+1}}\right)\right]\right.$$

$$+ e^{-(u^2+u)4\sqrt{\beta}}\left[1 - \Phi\,[u(\sqrt{4\beta}-1)\;-1]\right]$$

$$\left. - e^{+(u^2+u)4\sqrt{\beta}}\left[1 - \Phi\,[u(\sqrt{4\beta}+1)\;+1]\right]\right\} \tag{A-11}$$

where $u \equiv u_{LB}$ in Eqs.(A-10) and (A-11).

APPENDIX B

EVALUATION OF THE INTEGRAL FOR COSMIC GAMMA RAY PRODUCTION

Consider the evaluation of the cosmic gamma ray flux

$$j_\gamma = \frac{A}{\omega_0^\zeta} \int_0^{\frac{1}{2H}} dr \, \frac{e^{-k_{\gamma\gamma}^{ee} r}}{(1 + Hr)^{1+\zeta}} \; . \tag{B-1}$$

Let

$$\frac{k_{\gamma\gamma}^{ee}}{H} = \delta \qquad 1 + Hr = z$$

$$r = \frac{1}{H}(z - 1) \quad ; \quad \alpha = 1 + \zeta \; .$$

Then we can write Eq. (B-1) as

$$j_\gamma = \frac{A}{\omega_0^\zeta} \int_1^{\frac{3}{2}} \frac{dz}{H} \, \frac{e^{\delta - \delta z}}{z^\alpha} \; . \tag{B-2}$$

We can evaluate Eq. (B-1) by considering

$$I = \int \frac{dz \; e^{-\delta z}}{z^\alpha} \; . \tag{B-3}$$

An expansion by parts leads to

$$\int \frac{e^{-\delta z}}{z^{\alpha}}\, dz = \frac{-1}{(\alpha-1)} \frac{e^{-\delta z}}{z^{\alpha-1}} + \frac{\delta}{(\alpha-1)(\alpha-2)} \frac{e^{-\delta z}}{z^{\alpha-2}} + \frac{\delta^{2}}{(\alpha-1)(\alpha-2)} \int \frac{e^{-\delta z}}{z^{\alpha-2}} \, . \tag{B-4}$$

Because $\zeta \approx 3$ and $\alpha = 4$, we get

$$I = \frac{-1}{3} \frac{e^{-\delta z}}{z^{3}} + \frac{\delta}{6} \frac{e^{-\delta z}}{z^{2}} - \frac{\delta^{2}}{6} \frac{e^{-\delta z}}{z} - \frac{\delta^{3}}{6} \int_{1}^{\frac{3}{2}} \frac{e^{-\delta z}}{z}\, dz \, . \tag{B-5}$$

The "exponential integral" is usually defined as

$$\mathcal{E}i(p) = - \int_{p}^{\infty} \frac{e^{t}}{t}\, dt \, , \tag{B-6}$$

which we may write as

$$\mathcal{E}i(p) = - \int_{-p}^{-\infty} e^{-t} \frac{dt}{t}$$

or

$$\mathcal{E}i(-p) = \int_{-\infty}^{p} e^{-t} \frac{dt}{t} \, . \tag{B-7}$$

Consequently, if we write

$$\int_{1}^{\frac{3}{2}} \frac{e^{-\delta z}}{z}\, dz = \int_{\delta}^{\frac{3\delta}{2}} \frac{e^{-t}}{t}\, dt ,$$

we can write

$$\int_{1}^{\frac{3}{2}} \frac{e^{-\delta z}}{z}\, dz = \int_{-\infty}^{\frac{3\delta}{2}} \frac{e^{-t}}{t}\, dt - \int_{-\infty}^{\delta} \frac{e^{-t}}{t}\, dt$$

or

$$\int_{1}^{\frac{3}{2}} \frac{e^{-\delta z}}{z}\, dz = \mathcal{E}i\left(\frac{-3\delta}{2}\right) - \mathcal{E}i(-\delta) \tag{B-8}$$

For the situation where $\delta \rightarrow 0$, there is a removable singularity introduced by the definition, Eq. (B-8). We can remove this singularity in Eq. (B-8) by defining

$$\int_{1}^{\frac{3}{2}} \frac{e^{-\delta z}}{z}\, dz \xrightarrow[\delta \rightarrow 0]{} \int_{1}^{\frac{3}{2}} \frac{1}{z}\, dz = \ell n\, \frac{3}{2}$$

i.e.,

$$\mathcal{E}i\left(\frac{-3\delta}{2}\right) - \mathcal{E}i(-\delta) \equiv \ell n\, \frac{3}{2} \quad \text{at} \quad \delta = 0 .$$

We may now write Eq. (B-4) as

$$\int_1^{\frac{3}{2}} \frac{e^{-\delta z}}{z^4}\, dz = \left[\frac{-1}{3}\frac{e^{-\delta z}}{z^3} + \frac{\delta}{6}\frac{e^{-\delta z}}{z^2} - \frac{\delta^2}{6}\frac{e^{-\delta z}}{z}\right]\Bigg|_1^{\frac{3}{2}} - \frac{\delta^3}{6}\left\{\mathcal{E}i\left(\frac{-3\delta}{2}\right) - \mathcal{E}i(-\delta)\right\}$$

Consequently, the cosmic gamma ray flux is

$$j_\gamma = \frac{A}{\omega_0^\zeta} \cdot \frac{e^\delta}{H}\left\{\frac{e^{-\delta}}{3}\left[1 - \frac{8}{27}e^{-\delta/2}\right] - \frac{\delta e^{-\delta}}{6}\left[1 - \frac{4}{9}e^{-\delta/2}\right]\right.$$

$$\left. + \frac{\delta^2 e^{-\delta}}{6}\left[1 - \frac{2}{3}e^{-\delta/2}\right] + \frac{\delta^3}{6}\left[\left(-\mathcal{E}i\left(\frac{-3\delta}{2}\right)\right) - \left(-\mathcal{E}i(-\delta)\right)\right]\right\} \tag{B-9}$$

The predictions for our simple model for gamma ray production and attenuation throughout the universe depend upon the values of the astronomical constants involved. If we consider the local photon density for thermal photons in space to be of the order of 0.3 ev cm^{-3} (Ref. 43), it is sensible to consider

$$\delta = \frac{k_{\gamma\gamma}^{ee}}{H} \sim 1 .$$

Then we will predict, for gamma rays with energy $< 10^{14}$ ev,

$$j_\gamma(E_\gamma) = \frac{0.196}{H} \times \frac{A}{\omega_0^3} . \tag{B-10}$$

The alternate choice for the local photon density would be $n_{ph} \sim 10^{-3}$ ev cm^{-3}. We may now consider $\delta \sim 0$, whence we find

$$j_\gamma(E_\gamma) = \frac{0.235}{H} \times \frac{A}{\omega_0^3} \tag{B-11}$$

for gamma rays with energy $< 10^{14}$ ev. For energies above the region 10^{14} to 10^{18} ev, we may take $\delta \sim 0$.

APPENDIX C

EVALUATION OF INTEGRALS IN GAMMA RAY PRODUCTION SPECTRUM

An integral whose solution is important in the determination of the gamma ray spectrum is

$$I = \int_0^b \frac{dx}{x^2} e^{-(cx)^2 - (\frac{f}{x})^2} \tag{C-1}$$

The solution of Eq. (C-1), given below, has been suggested by N. Greenspan of Grumman. Let us first define

$$x = \frac{1}{v} ;$$

then

$$dx = - \frac{1}{v^2} dv = - x^2 dv .$$

Equation (C-1) should then be written as

$$I = - \int_{\frac{1}{a}}^{\frac{1}{b}} dv \; e^{-(\frac{c}{v})^2 - (fv)^2} . \tag{C-2}$$

We may now let

$$z = \frac{c}{v} + fv \tag{C-3}$$

$\therefore$

$$(\frac{c}{v})^2 + (fv)^2 = z^2 - 2cf$$

and

$$dv = \frac{1}{f}\,dz + \frac{c}{fv^2}\,dv \ .$$

Consequently, with these substitutions we may write Eq. (C-2) as

$$I = \frac{1}{f}\int_{z_b^+}^{z_a^+} dz\ e^{-z^2+2cf} - \frac{c}{f}\int_{\frac{1}{a}}^{\frac{1}{b}} \frac{dv}{v^2}\ e^{-\left(\frac{c}{v}\right)^2-(fv)^2} \qquad \text{(C-4)}$$

where

$$z_b^+ = cb + \frac{f}{b} \quad ; \quad z_a^+ = ca + \frac{f}{a} \ .$$

We next define

$$x = \frac{1}{v}$$

and

$$z = \frac{c}{v} - fv \ , \qquad \text{(C-5)}$$

giving limits

$$z_a^- = ca - \frac{f}{a}$$

$$z_b^- = cb - \frac{f}{b} \ .$$

Consequently, we may evaluate Eq. (C-1) as

$$I = \frac{-1}{f} \int_{z_b^-}^{z_a^-} dz\, e^{-z^2 - 2cf} + \frac{c}{f} \int_{\frac{1}{a}}^{\frac{1}{b}} \frac{dv}{v^2}\, e^{-\left(\frac{c}{v}\right)^2 - (fv)^2} \qquad \text{(C-6)}$$

Adding Eqs. (C-4) and (C-6) we find

$$I = \frac{e^{2cf}}{2f} \int_{z_b^+}^{z_a^+} e^{-z^2}\, dz - \frac{e^{-2cf}}{2f} \int_{z_b^-}^{z_a^-} e^{-z^2}\, dz\ , \qquad \text{(C-7)}$$

but

$$\Phi(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-z^2}\, dz\ .$$

Consequently,

$$I = \frac{\sqrt{\pi}}{4f} \left\{ e^{2cf} \left[\Phi(z_a^+) - \Phi(z_b^+) \right] - e^{-2cf} \left[\Phi(z_a^-) - \Phi(z_b^-) \right] \right\}\ . \qquad \text{(C-8)}$$

QED

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